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Automatic Control Laboratory

**Master Thesis**

# Social Welfare Optimization and Incentive Compatibility in Stochastic Electricity Markets with Renewable Integration

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# Abstract

As the integration of intermittent renewable energy resources such as wind and solar accelerates, traditional electricity markets, which require deterministic day-ahead bids from all participants, will become increasingly less efficient. This thesis addresses this challenge by proposing Stochastic Electricity Markets-a day-ahead electricity market in which renewable energy producers submit probability distribution bids rather than deterministic guesses. Stochastic Electricity Markets maximize expected social welfare and therefore provide smarter day-ahead scheduling and fairer pricing than deterministic markets. We demonstrate that Stochastic Electricity Markets ensure nonnegative expected/long-run cost recovery for producers in the aggregate day-ahead and real-time market, even though they may experience temporary losses in the day-ahead market. Furthermore, we investigate incentive compatibility for monopoly intermittent energy bidders, providing insight into when inventive compatibility fails, highlighting the need for market competition.

# Contents

# Notation

| Notation [1] | Definition |
|---|---|
| $D$ | Inelastic demand |
| $a^{\mathrm{d}}, a^{\mathrm{s}}$ | Day-ahead bid for selling |
| $p^{\mathrm{d}}, p^{\mathrm{s}}$ | Real-time bid for buying (penalty) |
| $r^{\mathrm{d}}, r^{\mathrm{s}}$ | Real-time bid for selling (reward) |
| $\Delta a^{\mathrm{d},+}$ | Upward incremental bid price of the TPP |
| $\Delta a^{\mathrm{d},-}$ | Downward incremental bid price of the TPP |
| $c$ | Optimal shortfall likelihood |
| $q^{\mathrm{d}}, q^{\mathrm{s}}$ | Day-ahead dispatch (day-ahead decision variables) |
| $\delta^{\mathrm{d}}, \delta^{\mathrm{s}}$ | Real-time dispatch (real-time decision variables) |
| $\omega$ | Actual realization of production of WPP |
| $Q^{\mathrm{s}}$ | WPP production estimation for the deterministic day-ahead market |
| $\lambda^{\mathrm{DA}}$ | Day-ahead market price |
| $\lambda^{\mathrm{RT}}$ | Real-time market price |
| $\mathcal{F}$ | Distribution bid submitted by the WPP |
| $\tilde{\mathcal{F}}$ | True production distribution of the WPP |
| $\pi(\cdot), \tilde{\pi}(\cdot)$ | Probability density function (PDF) of $\mathcal{F}$ and $\tilde{\mathcal{F}}$ |
| $\phi(\cdot), \tilde{\phi}(\cdot)$ | Cumulative distribution function (CDF) of $\mathcal{F}$ and $\tilde{\mathcal{F}}$ |
| $C^{\mathrm{DA}}$ | Optimal day-ahead production cost of the deterministic market |
| $C^{\mathrm{RT}}$ | Optimal real-time production cost |
| $C$ | Market operator's optimal expected total social production cost (following $\mathcal{F}$) |
| $\tilde{C}$ | True optimal expected total social production cost (following $\tilde{\mathcal{F}}$) |
| $P_1^{\mathrm{d}}, P_1^{\mathrm{s}}$ | Day-ahead profit |
| $P_2^{\mathrm{d}}, P_2^{\mathrm{s}}$ | Expected real-time profit |
| $P^{\mathrm{d}}, P^{\mathrm{s}}$ | Expected total profit |

[1]Superscript $\square^{\mathrm{d}}$: deterministic production (TPP); $\square^{\mathrm{s}}$: stochastic production (WPP)

# Chapter 1

# Introduction

## 1.1 Background

Electricity is fundamental to modern society, powering industries, businesses, and daily life. Historically, electricity generation, transmission, and distribution were often managed by vertically integrated utilities, where a single utility (e.g., private companies and government agencies) controlled the entire supply chain and set prices. In recent decades, many countries and regions have deregulated their electricity markets. This shift introduced competition with the goal of improving market efficiency and enabling more transparent, market-driven pricing mechanisms [1].

### 1.1.1 Organization of Electricity Markets

Electricity markets serve as platforms where various participants interact and agree on transactions for electricity. The key players in electricity markets are energy producers, consumers, and the market operator:

- **Producers**: Producers, utilizing various technologies, generate electricity and submit price bids to sell it on the market. These technologies can range from traditional sources like thermal and nuclear power to renewable sources such as wind and solar power, each with different production costs. Producers aim to maximize profits by selling electricity at prices higher than their marginal costs. Such profits are often referred to as *producers' surplus*.
- **Consumers**: Including individual users and large industrial companies, the consumers submit bits to purchase electricity on the electricity markets for their own use. Their goal is to maximize the utility, which is the difference between the market price and their willingness to pay. This utility is referred to as *consumers' surplus*. The willingness to pay can vary widely among consumers depending on their needs and cost considerations.
- **Market Operator (MO)**: The MO is the party that manages the market. It collects bids from producers and consumers, and determines the electricity dispatches and market price based on a set of published rules. The MO's primary goal is to optimize social welfare by balancing supply, demand, and prices in a fair and efficient manner.

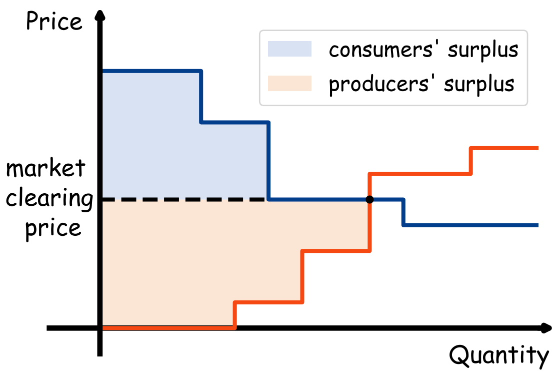
Apart from the mentioned participants, there are more parties involved in electricity markets, such as transmission system operators, distribution system operators, retailers, balancing authorities, and even financial speculators. We refer to [2] for further details about the composition of electricity markets.

While participants can engage in long-term electricity transactions via futures contracts, short-term interactions occur within the *electricity pool*. Although the operational details of the electricity pool may vary by region, they generally follow a similar structure. The electricity pool typically consists of two stages: the day-ahead (DA) market and the real-time (RT) market, both of which are organized as auctions [3]. The DA market operates one day prior to energy dispatch based on supply and demand forecasts, creating an optimal generation plan. The RT market, on the other hand, runs just minutes before energy delivery. It adjusts for any deviations from the DA plan, addressing uncertainties of both the production and demand. This real-time balancing shows significant importance for stochastic producers like a wind power producer, as their accurate production is usually not accessible before the closure of the DA market. Because of its role in balancing real-time supply and demand, the RT market is sometimes referred to as *the balancing market*. Some regions also implement intermediate markets between DA and RT markets to provide further flexibility and risk management.

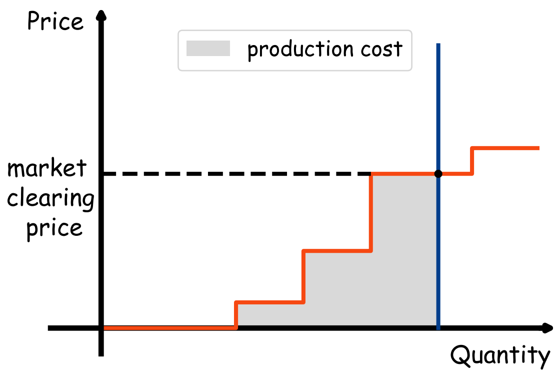
In both markets, producers and consumers submit price bids to sell or buy specified amounts of electricity at particular prices. The MO aggregates these bids to form *merit order curves*. More specifically, the MO arranges producers' (consumers') bids into a curve in ascending (descending) order of price. These two curves represent *supply curve* and *demand curve* of the market, whose intersection determines the *market clearing price*. This intersection is often referred to as the *market equilibrium*.

At this equilibrium, *social surplus*, i.e., the combined benefits to both producers and consumers, is maximized. The MO's primary objective is to achieve this equilibrium, ensuring market efficiency while respecting the balance between supply and demand.

The demand for electricity is typically highly *inelastic*. In other words, changes in the market price have minimal effects on the demand. Hence, sometimes the demand is set to a constant load forecast, and the demand curve is represented as a vertical line at this value. In this case, calculating social surplus is not feasible. Instead, the MO focuses on minimizing the total production cost as an alternative. Both elastic and inelastic cases are illustrated in Figure 1.1.




(a) Elastic Demand




(b) Inelastic Demand

Figure 1.1: Merit order curves constructed from producers' and consumers' bids

### 1.1.2 Desirable Properties

An efficiently functioning electricity market typically exhibits several key desirable properties, which ensure that the market operates fairly, transparently, and sustainably for all participants.

- **Maximizing Social Welfare**: Social welfare represents the total economic benefit to both producers and consumers. In the context of markets, social surplus always serves as the metric of social welfare. In an efficient market, social welfare is maximized at the market equilibrium, ensuring that the total combined benefits to society are as large as possible.

- **Incentive Compatibility**: An incentive-compatible market requires all participants to have an incentive to act truthfully. This means that producers should bid based on their actual production costs, and consumers should bid according to their true willingness to pay. Incentive compatibility ensures that market prices reflect the real value of electricity, promoting fairness and transparency. A well-known result reveals that a perfectly competitive market enjoys incentive compatibility.
- **Cost Recovery**: Cost recovery means that producers are able to recover their operating costs and earn at least zero or positive profits from participating in the electricity market. Ensuring cost recovery provides producers with the economic incentive to remain active in the market, which is crucial for maintaining a reliable electricity supply. If producers consistently fail to recover their costs, they may exit the market, reducing competition and threatening supply security.
- **Revenue Adequacy**: The MO handles several financial transactions, collecting payments from consumers and paying producers for the electricity they generate. In some cases, additional payments may be made to certain producers to ensure cost recovery such that they continue participating in the market [4]. Revenue adequacy requires the MO to balance these transactions, ensuring that its total revenues are sufficient to cover all payments without incurring a financial deficit [4].

### 1.1.3 Challenges under High Renewable Penetration

Driven by growing concerns over climate change, energy security, and sustainability, countries around the world are increasingly shifting towards renewable energy sources, implementing policies and investing in technologies to reduce carbon emissions and transition to cleaner, more resilient energy systems. For instance, in its 2022 strategy report, Denmark's national transmission system operator, Energinet[1], forecasted that "from 2022 to 2030, the installed volume of renewable energy in Denmark will quadruple" [5].

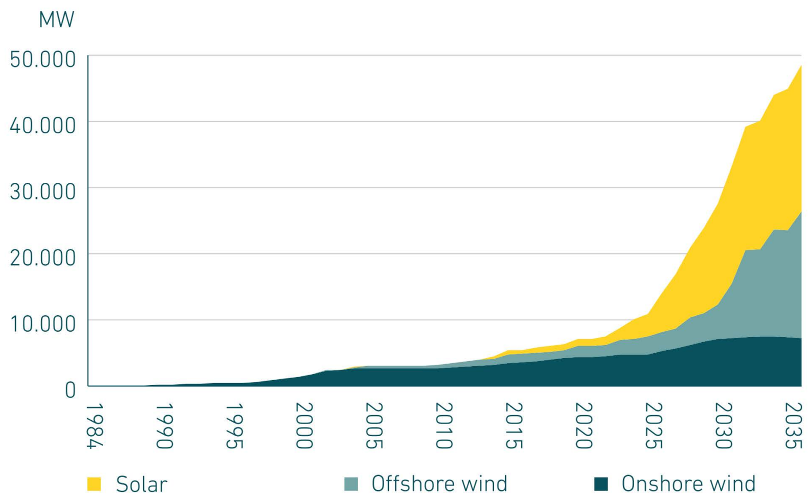



Figure 1.2: Expected renewable energy installation in Denmark [5]

Represented by solar and wind power, renewable energy technologies offer several key benefits. First, they are both environmentally friendly, producing electricity with a negligible carbon footprint compared to conventional fossil fuel-based generation. Moreover, they often come with lower marginal production costs. As a result, increasing the renewable integration tends to lower

[1]https://en.energinet.dk/

the overall production cost as well as the market price, as renewable energy typically shifts the supply curve to the right (see Figure 1.3).

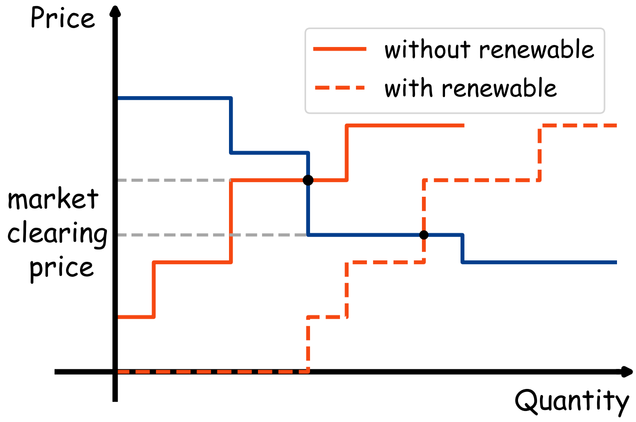



Figure 1.3: Cheap renewable energy resources typically shift right the merit-order supply curve and lead to a lower market clearing price.

However, renewable energy sources are often subject to significant production uncertainties as they are constrained by the availability of natural resources. Despite the emergence of advanced forecasting techniques [3, Chapter 2], accurately predicting generation quantities prior to energy dispatch remains challenging. More reliable production information is typically only available shortly before energy dispatch. Under such production uncertainties, the DA market scheduling can be less efficient, and more efforts are required in the RT market to balance production and consumption. Hence, it can lead to higher overall production costs and reduce total social welfare.

As renewable penetration continues to rise, the electricity market needs to handle the complexities associated with integrating large volumes of intermittent energy. Developing robust market mechanisms and technological solutions to manage this transition is crucial to ensure the economic and environmental benefits of renewable energy.

## 1.2 Related Work

To deal with increasing renewable penetration, many new market schemes have been proposed in the literature.

The most commonly used approach to promote higher social welfare is stochastic optimization (SO) [4, 6–10]. By modeling production uncertainties through stochastic distributions, SO typically maximizes the expected total social welfare in the DA market, which includes both the deterministic component (the DA market) and an additional stochastic component (the RT market). This approach allows DA market scheduling to account for the potential impact of decisions on the RT market.

For the sake of tractable computation, all these works characterize the production uncertainty using *scenarios* [3, Chapter 2.5.4], i.e., a finite set of pre-generated production realizations. These scenarios act as a discrete approximation of the actual uncertainty distribution, allowing for an approximated yet realistic representation of uncertainty in the optimization process.

Apart from optimal expected social welfare, many works also discuss cost recovery and revenue adequacy. The proposed markets in [6] and [7] successfully achieve cost recovery and revenue adequacy in expectation. Achieving revenue adequacy and cost recovery in expectation but not by scenario can be problematic if some producers are risk-averse. For example, a cautious producer might exit the market if they fail to make profits in certain scenarios. To address this issue, [4] extends the work of [6] to achieve scenario-wise cost recovery via uplift payments (where the MO covers the cost if producers incur losses) while maintaining revenue adequacy in expectation. This study also discusses the price distortion between the DA and RT prices. [9] proposes a bilevel market formulation, introducing an extra variable in the lower level (DA) problem to limit the DA dispatch for stochastic producers, with its value determined by the upper level (RT) problem. This also achieves scenario-wise cost recovery. On the other hand, [10] formulates an equilibrium model where each party (producers and the transmission operator) maximizes their probability-weighted profit for each individual scenario. This model achieves both revenue adequacy and cost recovery by scenario but at the cost of decreased social welfare.

Scenario-based SO involves a trade-off between closer distribution approximation and computational complexity. While a larger scenario set may better represent the uncertainty, it increases the complexity of the optimization problem and may still be insufficient. Recent works have explored the use of continuous distributions and chance constraints for market design.

[11] and [12] propose an expected social welfare optimization formulation for the DA market, where the uncertainty from stochastic energy production is assumed to follow a Gaussian distribution. Chance constraints are applied to production limitations. Rather than using the RT market to balance the production uncertainties, in this market, traditional producers respond to stochastic production deviations (from mean production) using proportional reaction policies, with these proportional factors introduced as optimization variables in the DA market formulation. A traditional producer receives a higher reward if it takes responsibility for a larger portion of the imbalance. This reward is independent of actual renewable production and thus eliminates the need to analyze scenario-based cost recovery and revenue adequacy. However, it does not reflect real-time resource scarcity as the RT markets. In fact, how a general distribution rather than scenarios influences the standard two-stage electricity market (including a DA market and an RT market) has hardly been studied. [13] further extends this method by incorporating transmission constraints and costs based on optimal power flow.

To further address the potential mischaracterization of uncertainty, [14] and [15] explore distributional robustness. [14] defines a moment ambiguity set containing zero-mean Gaussian distributions with different covariance matrices and minimizes the expected production cost in the worst case. [15] generalizes the ambiguity set, formalizing a Wasserstein distributional robust chance-constrained optimization problem.

While various methods for estimating renewable production uncertainties have been introduced in the literature [3], the question of where this information should come from is not yet discussed. Renewable production uncertainties (captured by either scenarios or general distributions) can either be estimated by the MO using historical data or provided directly by the renewable energy producers themselves. For the MO, allowing producers to provide this information shifts the prediction burden from the MO to the producers, enabling the MO to focus on optimizing social welfare. Moreover, producers may also prefer to submit their own distributions, especially if they disagree with externally generated estimates. However, the key question is whether renewable producers will submit truthful distributions or manipulate the information to gain a financial advantage. This issue requires further investigation.

## 1.3 Contribution and Thesis Structure

In this thesis, we aim to further interpret the stochastic electricity market model, its pricing mechanisms, and the properties it could achieve.

First, rather than using scenarios to capture the uncertainties similar to most existing works, we consider **general probability distributions** and study how different distributions influence optimal dispatches and market prices. Different from determining the proportional policies as a response to uncertainties as in [11, 12], we consider the standard RT market.

Second, while many recent works proposed new market models that promote scenario-wise revenue adequacy and cost recovery to incentivize the participation of risk-averse producers by compromising expected social welfare, in this thesis, we come back to the original market structure that **ensures the optimization of expected social welfare** and **cost recovery in expectation**. However, we argue that producers with foresight will be incentivized to participate in such a market. Thus, if we are able to convince producers to be foresighted, there is no need to focus on scenario-wise cost recovery. Notably, revenue adequacy is not the emphasis of this thesis as in our model, the MO only makes financial transactions regarding energy dispatches and thus naturally ensures revenue adequacy.

We also treat renewable energy producers as *distribution bidders* who submit their production distributions to the MO and study their **incentive compatibility**.

The remainder of this thesis is structured as follows: Chapter 2 introduces the problem settings and the standard deterministic market models. Chapter 3 studies expected social welfare optimization in the foresighted stochastic electricity market under stochastic productions and provides insights into the optimal dispatches and the pricing rule. Chapter 4 discusses further market properties including cost recovery to all the producers and the incentive compatibility of distribution bidders. Last but not least, we conclude the thesis in Chapter 5.

# Chapter 2

# The Deterministic Electricity Market

## 2.1 Problem Settings

Consider an electricity market involving *two energy producers*: a thermal power producer (TPP) and a wind power producer (WPP), with an *inelastic* demand $D \in \mathbb{R}_{\geq 0}$. The TPP represents traditional, dispatchable electricity producers and is assumed to have no upper limit on its production capacity. The WPP, representing renewable energy producers, exhibits stochastic production, meaning its actual output follows a probability distribution, denoted by $\omega \sim \mathcal{F}$. We treat it as the *only source of uncertainty* in the electricity market.

Through electricity auctions where producers submit price bids, the MO minimizes the social production cost and determines the energy dispatches. We assume all participants bid with *constant* prices.

## 2.2 Market Models

In this section, we first introduce the standard deterministic electricity market, which comprises two stages: a DA market and an RT market. We then make additional assumptions and provide a detailed discussion to justify their validity.

### 2.2.1 The DA Market

In a typical deterministic DA electricity market, the MO determines the dispatches based on the bid prices of the energy producers and the estimated WPP production quantity $Q^{\mathrm{s}} \in \mathbb{R}_{\geq 0}$. This is formulated as the following social production cost optimization problem:

$$C^{\mathrm{DA}} = \min_{q^{\mathrm{d}}, q^{\mathrm{s}}} \quad a^{\mathrm{d}} q^{\mathrm{d}} + a^{\mathrm{s}} q^{\mathrm{s}} \tag{2.1a}$$

$$\text{s.t.} \quad q^{\mathrm{d}} + q^{\mathrm{s}} = D, \quad (\lambda^{\mathrm{DA}}) \tag{2.1b}$$

$$0 \leq q^{\mathrm{d}} \tag{2.1c}$$

$$0 \leq q^{\mathrm{s}} \leq Q^{\mathrm{s}} \tag{2.1d}$$

(2.1a) captures the social production cost. (2.1b) is the demand balancing condition, whose corresponding dual variable $\lambda^{\mathrm{DA}}$ is equivalent to the market price. As we assume a very large TPP production capacity, (2.1c) only ensures nonnegativity for the TPP dispatch, while the WPP dispatch is additionally limited by the estimated quantity $Q^{\mathrm{s}}$ in (2.1d).

One reasonable choice for the estimated production $Q^{\mathrm{s}}$ is the mean value of the production distribution $\mathcal{F}$. For risk-averse cases, $Q^{\mathrm{s}}$ can be selected based on a specific quantile. For example,

$Q^{\mathrm{s}} = \max_q\{\Pr\{\omega \geq q\} \geq \epsilon\}$, where $\epsilon \in [0,1]$ represents the confidence level for producing at least $Q^{\mathrm{s}}$.

### 2.2.2 The RT Market

Following the DA market, a RT market is operated to balance supply and demand over a shorter time horizon. Due to the short time base, the exact WPP energy production $\omega$ is considered known in the RT market rather than treated as uncertain as in the DA market. Each producer is allowed to buy or sell additional energy in the RT market with a buying price $p$ and a selling price $r$. The RT dispatches $\delta^{\mathrm{d}}$ and $\delta^{\mathrm{s}}$ are viewed as a revision of the DA decisions $q^{\mathrm{d}}$ and $q^{\mathrm{s}}$, under the revelation of the actual WPP production. A positive RT dispatch represents an extra production. Conversely, a negative RT dispatch means that the producer buys some (cheap) energy from others to allow a reduction in its own production.

The MO minimizes the RT social production cost following the optimization problem in (2.2).

$$C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) = \min_{\delta^{\mathrm{d}},\delta^{\mathrm{s}}} \quad p^{\mathrm{d}}[\delta^{\mathrm{d}}]_+ - r^{\mathrm{d}}[\delta^{\mathrm{d}}]_- + p^{\mathrm{s}}[\delta^{\mathrm{s}}]_+ - r^{\mathrm{s}}[\delta^{\mathrm{s}}]_- \tag{2.2a}$$

$$\text{s.t.} \quad \delta^{\mathrm{d}} + \delta^{\mathrm{s}} = 0, \quad (\lambda^{\mathrm{RT}}) \tag{2.2b}$$

$$0 \leq q^{\mathrm{d}} + \delta^{\mathrm{d}}. \tag{2.2c}$$

$$0 \leq q^{\mathrm{s}} + \delta^{\mathrm{s}} \leq \omega \tag{2.2d}$$

Similar to the DA market, (2.2b) is the balancing condition. $\lambda^{\mathrm{RT}}$, the corresponding dual variable is the RT market price. $q^{\mathrm{d}} + \delta^{\mathrm{d}}$ and $q^{\mathrm{s}} + \delta^{\mathrm{s}}$ represent the total dispatches for both producers. (2.2c) ensures that the total TPP dispatch is nonnegative, while (2.2d) limits the upper bound of the total WPP dispatch. Note that while the upper bound for WPP dispatch in the DA market is based on production estimates $Q^{\mathrm{s}}$, the RT market operates with full information on the actual production quantity of WPP $\omega$.

The objective function (2.2a) is the RT social production cost, where $[x]_+ = \max(x, 0)$ (positive part) and $[x]_- = \max(-x, 0)$ (negative part) for all $x \in \mathbb{R}$. Extra production $[\delta^{\mathrm{d}}]_+$ and $[\delta^{\mathrm{s}}]_+$ incur additional production costs, whereas a reduction of production $[\delta^{\mathrm{d}}]_-$ and $[\delta^{\mathrm{s}}]_-$ recover certain costs from the DA schedule.

This setting allows the market to amend for the consequences of the potential mismatch between the estimated production $Q^{\mathrm{s}}$ and the actual production $\omega$. The MO can ask the TPP to produce additional energy to amend for a shortfall of the WPP. When the WPP achieves a production surplus, the scheme also allows the TPP to buy cheap energy from the WPP as a replacement for producing itself to achieve a lower social production cost.

### 2.2.3 Assumptions and Discussion

One may perceive the TPP as the aggregation of a population of traditional energy producers within a perfect competitive electricity market, where all producers bid truthfully. The WPP in the setting is an extra producer introduced to this competitive market. Hence, we assume the truthfulness of the TPP's bids in Assumption 1.

**Assumption 1.** *The TPP submits truthful bid prices. More specifically, the selling prices, $a^{\mathrm{d}}$ and $p^{\mathrm{d}}$, reflect its marginal production costs in the DA and RT markets respectively, and the buying price $r^{\mathrm{d}}$ represents its true willingness to buy on the RT market.*

Since renewable energy producers typically exhibit very low marginal production cost, in this work, we assume zero marginal production cost for the WPP as in Assumption 2.

**Assumption 2.** *The WPP exhibits zero marginal production cost such that the bid prices* $a^{\mathrm{s}} = p^{\mathrm{s}} = r^{\mathrm{s}} = 0$.

We then focus on the relationship among the DA selling price $a^{\mathrm{d}}$, the RT selling price $p^{\mathrm{d}}$, and the RT buying price $r^{\mathrm{d}}$, of the TPP.

**Definition 2.2.1** (Incremental Bid Prices [4])**.** *We define the upward and downward incremental bid prices of the TPP as* $\Delta a^{\mathrm{d},+} := p^{\mathrm{d}} - a^{\mathrm{d}}$ *and* $\Delta a^{\mathrm{d},-} := a^{\mathrm{d}} - r^{\mathrm{d}}$.

**Assumption 3.** *The TPP admits nonnegative incremental bid prices, i.e.,* $\Delta a^{\mathrm{d},+} \geq 0, \Delta a^{\mathrm{d},-} \geq 0$.

Assumption 3 is generally reasonable due to several economic and operational factors. Additional TPP production typically falls on the right-hand side of the merit-order curve, meaning that the marginal cost of extra production in the RT market is higher than what is in the DA market. Moreover, RT production generally requires flexible production units, which tend to incur higher marginal costs. Therefore, it is reasonable to assume that $p^{\mathrm{d}} \geq a^{\mathrm{d}}$ ($\Delta a^{\mathrm{d},+} \geq 0$).

On the other hand, $r^{\mathrm{d}}$, TPP's willingness to buy, reflects the highest price that the TPP would accept to purchase energy on the RT market, without losing profits from buying energy and reducing its scheduled production. When reducing its production, the TPP can only recover a portion of the DA marginal production cost $a^{\mathrm{d}}$. For example, 80% of $a^{\mathrm{d}}$ might cover ingredient costs (such as coal), and the remaining 20% counts for the labor costs. While the ingredient costs are recoverable, the TPP cannot recoup the labor costs by reducing production. As a result, the TPP can only benefit from purchasing energy at a price no higher than $0.8a^{\mathrm{d}}$. Due to the potential presence of these unrecoverable costs, it is also reasonable to assume $a^{\mathrm{d}} \geq r^{\mathrm{d}}$ ($\Delta a^{\mathrm{d},-} \geq 0$).

For the rest of this thesis, we assume Assumption 1, 2, and 3 hold.

## 2.3 Closed-Form Solutions

We now solve the deterministic DA and RT markets in closed form under the mentioned Assumptions. We first focus on the DA market. As $a^{\mathrm{d}} > a^{\mathrm{s}} = 0$, the MO tends to make the dispatch to the WPP as much as possible, upper-bounded by the estimated WPP production $Q^{\mathrm{s}}$, to achieve lower social production cost. Following this logic, the solution to the deterministic DA market is rather trivial, as described in equation (2.3) and sketched in Figure 2.1.

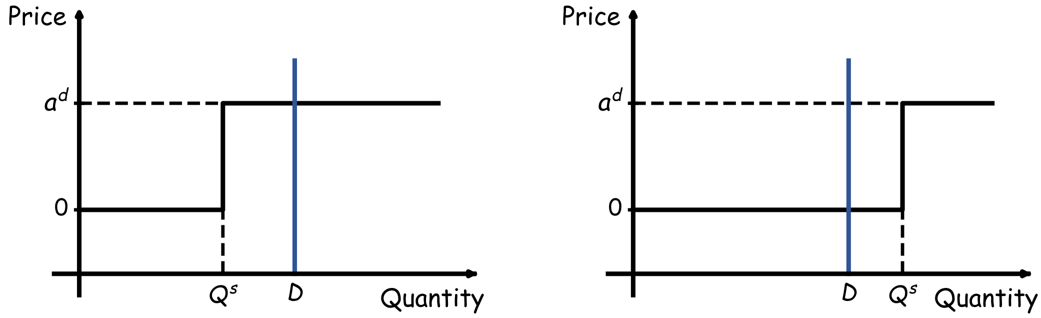
$$q^{\mathrm{s},*} = \begin{cases} Q^{\mathrm{s}}, & Q^{\mathrm{s}} < D \\ D, & Q^{\mathrm{s}} \geq D, \end{cases} \qquad q^{\mathrm{d},*} = D - q^{\mathrm{s},*}, \quad \lambda^{\mathrm{DA},*} = \begin{cases} a^{\mathrm{d}}, & Q^{\mathrm{s}} < D \\ 0, & Q^{\mathrm{s}} \geq D. \end{cases} \tag{2.3}$$

If the estimation suggests that the WPP can produce more than the demand ($Q^{\mathrm{s}} \geq D$), there is no incentive to dispatch any thermal power units, as the market is saturated with cheap energy. In this case, the optimal WPP dispatch is $q^{\mathrm{s},*} = D$, and the market price is decided by the marginal production cost of the WPP as $\lambda^{\mathrm{DA},*} = a^{\mathrm{s}} = 0$. Conversely, if the WPP is unable to meet the entire demand ($Q^{\mathrm{s}} < D$), the TPP must step in to cover the rest of the demand, resulting in the market price being set at the TPP's marginal production cost $a^{\mathrm{d}}$.

For the RT market (2.2), the optimal RT cost is

$$C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) = p^{\mathrm{d}} \left[\min(\omega - q^{\mathrm{s}}, q^{\mathrm{d}})\right]_{-} - r^{\mathrm{d}} \left[\min(\omega - q^{\mathrm{s}}, q^{\mathrm{d}})\right]_{+} \tag{2.4}$$

with the optimal RT dispatches being $\delta^{\mathrm{s},*} = -\delta^{\mathrm{d},*} = \min(\omega - q^{\mathrm{s}}, q^{\mathrm{d}})$. $\omega - q^{\mathrm{s}}$ characterizes the *surplus* (if positive) or *shortfall* (if negative) of the WPP production. The physical meaning of




(a) $Q^s < D$ (b) $Q^s > D$

Figure 2.1: The demand curve (blue) and the supply curve (black) for different amounts of energy the WPP is expected to produce

this solution is also trivial. If there exhibits a WPP shortfall with $\omega < q^s$, the TPP is required to produce extra $\delta^{d,*} = q^s - \omega$ to balance the demand. If $\omega > q^s$, i.e., the WPP produces a surplus, the MO tends to allow the TPP to buy $\omega - q^s$ amount of energy from the WPP and reduce its own production, since this reduces the overall social production cost. The DA TPP dispatch $q^d$, limits the maximum amount of energy the WPP can sell to the TPP on the RT market. The corresponding RT market price is

$$\lambda^{RT,*} = \begin{cases} p^d, & \omega \leq q^s \\ r^d, & q^s < \omega \leq q^d + q^s \\ 0, & q^d + q^s < \omega. \end{cases} \tag{2.5}$$

The price is set to $p^d$, reflecting the marginal production cost of the TPP in the RT market, when the TPP needs to generate additional electricity as compensation. And the WPP sells for $r^d$ (reflecting the willingness to pay) per unit of energy when there is a surplus but the overall production does not exceed the demand ($\omega <= q^d + q^s = D$). When the WPP produces too much, such that it exceeds the total demand and some of the energy is deserted, the price is 0.

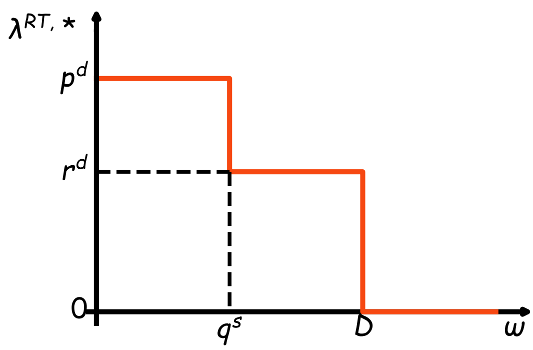



Figure 2.2: RT Market Price

# Chapter 3

# The Stochastic Electricity Market

## 3.1 Proactive DA Market

In the introduced deterministic market formulation, the MO makes DA decisions by optimizing the DA social cost $C^{\mathrm{DA}}$, relying on the WPP production estimation. However, the DA dispatches influence the RT cost $C^{\mathrm{RT}}$. This reveals the limitation of the deterministic model, as the optimal DA dispatches for the DA market might not minimize the *total social cost* of the DA and RT markets, i.e., $C^{\mathrm{DA}} + C^{\mathrm{RT}}$, especially with a skeptical estimation $Q^{\mathrm{s}}$.

To increase market efficiency with the integration of renewable energy, it is reasonable to take the impact of DA decisions on the RT market into consideration when determining the DA dispatches, such that we optimize the total social cost instead of only the DA social cost, as shown in problem (3.1).

$$
\begin{aligned}
C(\mathcal{F}) = \min_{q^{\mathrm{d}}, q^{\mathrm{s}}} \quad & a^{\mathrm{d}} q^{\mathrm{d}} + a^{\mathrm{s}} q^{\mathrm{s}} + \mathbb{E}^{\omega \sim \mathcal{F}} \left[ C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) \right] && (3.1\mathrm{a}) \\
\text{s.t.} \quad & q^{\mathrm{d}} + q^{\mathrm{s}} = D, \quad (\lambda^{\mathrm{DA}}) && (3.1\mathrm{b}) \\
& q^{\mathrm{d}}, q^{\mathrm{s}} \geq 0 && (3.1\mathrm{c})
\end{aligned}
$$

As the WPP has stochastic production, we introduce the *expected RT social cost* as an extra penalty to the DA market in the objective function (3.1a). $\lambda^{\mathrm{DA}}$ denotes the dual variable of the balancing condition (3.1b). Constraint (3.1c) omit the upper bound for the WPP dispatch compared to (2.1d). As in the deterministic DA market 2.1, the market price is set as $\lambda^{\mathrm{DA}}$.

Problem (3.1) admits *two-stage*. In particular, the RT market (2.2) acts as the second stage of the problem. Under Assumption 2, the solution to the second stage problem is given by (2.4) . We can thus rewrite the stochastic market (3.1) to a single-stage form as follows:

$$
\begin{aligned}
C(\mathcal{F}) = \min_{q^{\mathrm{d}}, q^{\mathrm{s}}} \quad & \mathbb{E}^{\omega \sim \mathcal{F}} \left[ \varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) \right] && (3.2\mathrm{a}) \\
\text{s.t.} \quad & q^{\mathrm{d}} + q^{\mathrm{s}} = D, \quad (\lambda^{\mathrm{DA}}) && (3.2\mathrm{b}) \\
& q^{\mathrm{d}}, q^{\mathrm{s}} \geq 0 && (3.2\mathrm{c})
\end{aligned}
$$

where

$$
\begin{aligned}
\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) &:= a^{\mathrm{d}} q^{\mathrm{d}} + C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) \\
&= a^{\mathrm{d}} q^{\mathrm{d}} + p^{\mathrm{d}} \left[ \min(\omega - q^{\mathrm{s}}, q^{\mathrm{d}}) \right]_{-} - r^{\mathrm{d}} \left[ \min(\omega - q^{\mathrm{s}}, q^{\mathrm{d}}) \right]_{+}.
\end{aligned} \tag{3.3}
$$

Equation (3.3) defines the total social cost with the DA dispatches $q^{\mathrm{d}}$ and $q^{\mathrm{s}}$, under a realization of the WPP's production $\omega$. Note that market (3.1) and market (3.2) are equivalent.

In the rest of this section, we show that problem 3.2 is convex and provides the closed-form solution to it.

**Theorem 3.1.1.** *The total social cost function* $\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)$ *is convex and hence, the expected total social cost* $\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]$ *(the objective of market* (3.2)*) is convex.*

*Proof.* Using $[q]_+ - [q]_- = q$, we rewrite the total social cost function as

$$\begin{aligned}\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) &= a^{\mathrm{d}}q^{\mathrm{d}} + (p^{\mathrm{d}} - r^{\mathrm{d}})\left[\min(\omega - q^{\mathrm{s}}, q^{\mathrm{d}})\right]_- - r^{\mathrm{d}}\min(\omega - q^{\mathrm{s}}, q^{\mathrm{d}}) \\ &= a^{\mathrm{d}}q^{\mathrm{d}} + (p^{\mathrm{d}} - r^{\mathrm{d}})\max(q^{\mathrm{s}} - \omega, -q^{\mathrm{d}}, 0) + r^{\mathrm{d}}\max(q^{\mathrm{s}} - \omega, -q^{\mathrm{d}}).\end{aligned} \tag{3.4}$$

The social cost function $\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)$ is the sum of several convex functions, making itself and furthermore, the expected total social cost function $\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]$, convex. □

**Theorem 3.1.2.** *Suppose the probability density function (PDF) of the WPP production distribution* $\pi(x)$ *is* ***continuous*** *in* $\mathbb{R}_{\geq 0}$*. Then the optimal DA dispatch and the corresponding DA market price of market* (3.2) *are given by*

$$q^{\mathrm{s},*}(\mathcal{F}) = \begin{cases} \phi^{-1}(c), & \phi(D) > c \\ D, & \phi(D) \leq c, \end{cases} \tag{3.5a}$$

$$q^{\mathrm{d},*}(\mathcal{F}) = D - q^{\mathrm{s},*}(\mathcal{F}), \tag{3.5b}$$

$$\lambda^{\mathrm{DA},*}(\mathcal{F}) = \begin{cases} a^{\mathrm{d}} - r^{\mathrm{d}}(1 - \phi(D)), & \phi(D) > c \\ p^{\mathrm{d}}\phi(D), & \phi(D) \leq c, \end{cases} \tag{3.5c}$$

*where* $c := \frac{\Delta a^{\mathrm{d},-}}{\Delta a^{\mathrm{d},+} + \Delta a^{\mathrm{d},-}} = \frac{a^{\mathrm{d}} - r^{\mathrm{d}}}{p^{\mathrm{d}} - r^{\mathrm{d}}}$.

*Proof.* The expected social cost can be expanded in detail as follows:

$$\begin{aligned}&\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] \\ &= a^{\mathrm{d}}q^{\mathrm{d}} + \int_0^{q^{\mathrm{s}}} p^{\mathrm{d}}(q^{\mathrm{s}} - \omega)\pi(\omega)\mathrm{d}\omega - \int_{q^{\mathrm{s}}}^{q^{\mathrm{d}}+q^{\mathrm{s}}} r^{\mathrm{d}}(\omega - q^{\mathrm{s}})\pi(\omega)\mathrm{d}\omega - \int_{q^{\mathrm{d}}+q^{\mathrm{s}}}^{+\infty} r^{\mathrm{d}}q^{\mathrm{d}}\pi(\omega)\mathrm{d}\omega \\ &= a^{\mathrm{d}}q^{\mathrm{d}} + \int_0^{q^{\mathrm{s}}} p^{\mathrm{d}}(q^{\mathrm{s}} - \omega)\pi(\omega)\mathrm{d}\omega - \int_{q^{\mathrm{s}}}^{q^{\mathrm{d}}+q^{\mathrm{s}}} r^{\mathrm{d}}(\omega - q^{\mathrm{s}})\pi(\omega)\mathrm{d}\omega - r^{\mathrm{d}}q^{\mathrm{d}}\left(1 - \phi(q^{\mathrm{d}} + q^{\mathrm{s}})\right)\end{aligned} \tag{3.6}$$

Since $\pi(x)$ is continuous, applying the *Leibniz Integral Rule*, we calculate the first-order derivatives of the objective function towards the decision variables (DA dispatches) as

$$\begin{aligned}\nabla_{q^{\mathrm{d}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] &= a^{\mathrm{d}} - r^{\mathrm{d}}\left(1 - \phi(q^{\mathrm{d}} + q^{\mathrm{s}})\right) \\ \nabla_{q^{\mathrm{s}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] &= (p^{\mathrm{d}} - r^{\mathrm{d}})\phi(q^{\mathrm{s}}) + r^{\mathrm{d}}\phi(q^{\mathrm{d}} + q^{\mathrm{s}})\end{aligned}$$

We then write down the Lagrangian function

$$\mathcal{L}(q^{\mathrm{d}}, q^{\mathrm{s}}, \lambda^{\mathrm{DA}}, \lambda^{\mathrm{d}}, \lambda^{\mathrm{s}}) = \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] + \lambda^{\mathrm{DA}}(D - q^{\mathrm{d}} - q^{\mathrm{s}}) - \lambda^{\mathrm{d}}q^{\mathrm{d}} - \lambda^{\mathrm{s}}q^{\mathrm{s}}$$

and list the Karush-Kuhn-Tucker (KKT) optimality conditions as follows:

$$\begin{aligned}\nabla_{q^{\mathrm{d}}}\mathcal{L} &= \nabla_{q^{\mathrm{d}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d},*},q^{\mathrm{s},*},\omega)\right] - \lambda^{\mathrm{DA},*} - \lambda^{\mathrm{d},*} \\ &= a^{\mathrm{d}} - r^{\mathrm{d}}\left(1-\phi(q^{\mathrm{d},*}+q^{\mathrm{s},*})\right) - \lambda^{\mathrm{DA},*} - \lambda^{\mathrm{d},*} = 0\end{aligned} \tag{3.7a}$$

$$\begin{aligned}\nabla_{q^{\mathrm{s}}}\mathcal{L} &= \nabla_{q^{\mathrm{s}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d},*},q^{\mathrm{s},*},\omega)\right] - \lambda^{\mathrm{DA},*} - \lambda^{\mathrm{s},*} \\ &= (p^{\mathrm{d}}-r^{\mathrm{d}})\phi(q^{\mathrm{s},*}) + r^{\mathrm{d}}\phi(q^{\mathrm{d},*}+q^{\mathrm{s},*}) - \lambda^{\mathrm{DA},*} - \lambda^{\mathrm{s},*} = 0\end{aligned} \tag{3.7b}$$

$$q^{\mathrm{d},*}+q^{\mathrm{s},*} = D \tag{3.7c}$$

$$0 \le q^{\mathrm{d},*} \perp \lambda^{\mathrm{d},*} \ge 0 \tag{3.7d}$$

$$0 \le q^{\mathrm{s},*} \perp \lambda^{\mathrm{s},*} \ge 0 \tag{3.7e}$$

Note that "$\perp$" denotes complementary slackness. The solution in (3.5) satisfies the KKT conditions. As market (3.2) is a convex optimization problem according to Theorem 3.1.1, this solution is optimal [16, Chapter 5.5]. □

The proof above requires continuity of the PDF $\pi(\cdot)$ to ensure smoothness of the objective function $\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}},q^{\mathrm{s}},\omega)\right]$, enabling the calculation of its gradients towards the decision variables. However, borrowing results from *nonsmooth convex analysis*, we can extend the result and show the optimality of (3.5) for *general distributions* that do not exhibit continuous $\pi(\cdot)$, such as discrete distributions and mixed distributions. Details and sidenotes about nonsmooth convex analysis can be found in Appendix A.

**Theorem 3.1.3.** *For a general WPP production distribution $\mathcal{F}$ supported on $\mathbb{R}_{\ge 0}$, the optimal DA dispatch and the corresponding DA market price to market* (3.2) *are given by* (3.5)*.*

*Proof.* Denote the decision variables by $x = \begin{bmatrix} q^{\mathrm{d}} & q^{\mathrm{s}}\end{bmatrix}^\top$. With a slight abuse of notations, in the remainder of this proof, we denote the total social cost function $\varphi(q^{\mathrm{d}},q^{\mathrm{s}},\omega)$ in (3.4) by $\varphi(x,\omega)$. For a general distribution $\mathcal{F}$, the expected social cost function $\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(x,\omega)\right]$ is convex (see Theorem 3.1.1), continuous, but not necessarily smooth, and thus not everywhere differentiable. Therefore, we use nonsmooth convex analysis to prove this theorem. More specifically, we show that the solution (3.5) satisfies the sufficient optimality condition stated in Lemma A.2.1. To do so, we first compute the normal cone (Definition A.1.5) to constraints (3.2b)-(3.2c) and the subdifferential (Definition A.1.1) of the expected social cost function (3.2a), and then prove that 0 lies in the Minkowski Sum of the subdifferential and the normal cone.

- **Step 1: Computing the Normal Cone**

Denote the feasible sets to three individual constraints in (3.2b)-(3.2c) respectively by

$$\begin{aligned} S^{\mathrm{DA}} &:= \left\{\begin{bmatrix} q^{\mathrm{d}} & q^{\mathrm{s}}\end{bmatrix}^\top \;\middle|\; q^{\mathrm{d}}+q^{\mathrm{s}} = D,\ q^{\mathrm{d}},q^{\mathrm{s}}\in\mathbb{R}\right\}, \\ S^{\mathrm{d}} &:= \left\{\begin{bmatrix} q^{\mathrm{d}} & q^{\mathrm{s}}\end{bmatrix}^\top \;\middle|\; q^{\mathrm{d}}\ge 0, q^{\mathrm{s}}\in\mathbb{R}\right\}, \\ S^{\mathrm{s}} &:= \left\{\begin{bmatrix} q^{\mathrm{d}} & q^{\mathrm{s}}\end{bmatrix}^\top \;\middle|\; q^{\mathrm{s}}\ge 0, q^{\mathrm{d}}\in\mathbb{R}\right\}. \end{aligned}$$

It is trivial to compute the normal cones to these three sets, which are listed below:

$$N_{S^{\mathrm{DA}}}(x) = \left\{\lambda^{\mathrm{DA}}\begin{bmatrix}-1 & -1\end{bmatrix}^\top \;\middle|\; \lambda^{\mathrm{DA}}\in\mathbb{R}\right\},\ \forall x\in S^{\mathrm{DA}}$$

$$N_{S^{\mathrm{d}}}(x) = \begin{cases}\{\lambda^{\mathrm{d}}\begin{bmatrix}-1 & 0\end{bmatrix}^\top \mid \lambda^{\mathrm{d}}\ge 0\}, & q^{\mathrm{d}}=0\\ 0, & q^{\mathrm{d}}>0\end{cases}$$

$$N_{S^{\mathrm{s}}}(x) = \begin{cases}\{\lambda^{\mathrm{s}}\begin{bmatrix}0 & -1\end{bmatrix}^\top \mid \lambda^{\mathrm{s}}\ge 0\}, & q^{\mathrm{s}}=0\\ 0, & q^{\mathrm{s}}>0\end{cases}$$

The feasible set to the entire problem (3.2) is given by the intersection of these three sets: $S := S^{\mathrm{DA}} \cap S^{\mathrm{d}} \cap S^{\mathrm{s}}$. According to the normal cone intersection rule in Lemma A.1.6, we can find a subset of the normal cone to all three constraints

$$N_S(x) \supset \hat{N}_S(x) = \begin{cases} \left\{\lambda^{\mathrm{s}} \begin{bmatrix}0 & -1\end{bmatrix}^\top + \lambda^{\mathrm{DA}} \begin{bmatrix}-1 & -1\end{bmatrix}^\top \,\middle|\, \lambda^{\mathrm{s}} \geq 0, \lambda^{\mathrm{DA}} \in \mathbb{R}\right\}, & x = \begin{bmatrix}D & 0\end{bmatrix}^\top \\ \left\{\lambda^{\mathrm{d}} \begin{bmatrix}-1 & 0\end{bmatrix}^\top + \lambda^{\mathrm{DA}} \begin{bmatrix}-1 & -1\end{bmatrix}^\top \,\middle|\, \lambda^{\mathrm{d}} \geq 0, \lambda^{\mathrm{DA}} \in \mathbb{R}\right\}, & x = \begin{bmatrix}0 & D\end{bmatrix}^\top \\ \left\{\lambda^{\mathrm{DA}} \begin{bmatrix}-1 & -1\end{bmatrix}^\top \,\middle|\, \lambda^{\mathrm{DA}} \in \mathbb{R}\right\}, & \text{any other } x \in S. \end{cases}$$

- **Step 2: Computing the Subdifferential**

To find subgradients of the expected total social cost function $\mathbb{E}^{\omega\sim\mathcal{F}}[\varphi(x,\omega)]$, we first compute the subdifferential of $\varphi(x,\omega)$ under different realizations of the WPP production $\omega$. We use Lemma A.1.4 to deal with the pointwise maximum function within $\varphi(x,\omega)$. The sudifferential is listed in Table 3.1. Note that Conv $\{\cdot\}$ denotes the convex hull of the given elements.

Table 3.1: Subdifferential of the total social cost function $\varphi(x,\omega)$

| Case | $\omega \sim \mathcal{F}$ | Subdifferential $\partial_x\varphi(x,\omega)$ |
|---|---|---|
| 1 | $\omega - q^{\mathrm{s}} < 0 \leq q^{\mathrm{d}}$ | $\begin{bmatrix}a^{\mathrm{d}} & p^{\mathrm{d}}\end{bmatrix}^\top$ |
| 2 | $0 < \omega - q^{\mathrm{s}} < q^{\mathrm{d}}$ | $\begin{bmatrix}a^{\mathrm{d}} & r^{\mathrm{d}}\end{bmatrix}^\top$ |
| 3 | $0 < q^{\mathrm{d}} < \omega - q^{\mathrm{s}}$ | $\begin{bmatrix}a^{\mathrm{d}} - r^{\mathrm{d}} & 0\end{bmatrix}^\top$ |
| 4 | $0 = q^{\mathrm{d}} < \omega - q^{\mathrm{s}}$ | $\begin{bmatrix}a^{\mathrm{d}} - r^{\mathrm{d}} & 0\end{bmatrix}^\top + (p^{\mathrm{d}} - r^{\mathrm{d}})\mathrm{Conv}\left\{\begin{bmatrix}-1 & 0\end{bmatrix}^\top, \begin{bmatrix}0 & 0\end{bmatrix}^\top\right\}$ |
| 5 | $0 = \omega - q^{\mathrm{s}} < q^{\mathrm{d}}$ | $\begin{bmatrix}a^{\mathrm{d}} & r^{\mathrm{d}}\end{bmatrix}^\top + (p^{\mathrm{d}} - r^{\mathrm{d}})\mathrm{Conv}\left\{\begin{bmatrix}0 & 1\end{bmatrix}^\top, \begin{bmatrix}0 & 0\end{bmatrix}^\top\right\}$ |
| 6 | $0 < \omega - q^{\mathrm{s}} = q^{\mathrm{d}}$ | $\begin{bmatrix}a^{\mathrm{d}} & 0\end{bmatrix}^\top + r^{\mathrm{d}}\mathrm{Conv}\left\{\begin{bmatrix}-1 & 0\end{bmatrix}^\top, \begin{bmatrix}0 & 1\end{bmatrix}^\top\right\}$ |
| 7 | $0 = \omega - q^{\mathrm{s}} = q^{\mathrm{d}}$ | $\begin{bmatrix}a^{\mathrm{d}} & 0\end{bmatrix}^\top + \mathrm{Conv}\left\{\begin{bmatrix}-p^{\mathrm{d}} & 0\end{bmatrix}^\top, \begin{bmatrix}-r^{\mathrm{d}} & 0\end{bmatrix}^\top, \begin{bmatrix}0 & r^{\mathrm{d}}\end{bmatrix}^\top, \begin{bmatrix}0 & p^{\mathrm{d}}\end{bmatrix}^\top\right\}$ |

According to the weak subdifferential calculus rules for expectations presented in Proposition A.1.3, we are then able to construct a subgradient function $g(\omega) \in \partial_x\varphi(x,\omega), \forall\omega$ such that $\mathbb{E}^{\omega\sim\mathcal{F}}[g(\omega)] \in \partial_x\mathbb{E}^{\omega\sim\mathcal{F}}[\varphi(x,\omega)]$.

- **Step 3.1: Examining the Optimality Conditions ($\phi(D) > c$)**

We first examine if the candidate solution for $\phi(D) > c$ is optimal, which is $q^{\mathrm{s},*} = \phi^{-1}(c), q^{\mathrm{d},*} = D - \phi^{-1}(c), \lambda^{\mathrm{DA},*} = a^{\mathrm{d}} - r^{\mathrm{d}}(1 - \phi(D))$. If $q^{\mathrm{s},*} = \phi^{-1}(c) < D$, case 1, 2, 3, 5, and 6 in Table 3.1 could happen with the probability of $\phi(\phi^{-1}(c)) - \Pr\{\omega = \phi^{-1}(c)\}$, $\phi(D) - \phi(\phi^{-1}(c)) - \Pr\{\omega = D\}$, $1 - \phi(D)$, $\Pr\{\omega = \phi^{-1}(c)\}$, and $\Pr\{\omega = D\}$, respectively.

Let

$$k = \begin{cases} \frac{c - \phi(\phi^{-1}(c)) + \Pr\{\omega = \phi^{-1}(c)\}}{\Pr\{\omega = \phi^{-1}(c)\}}, & \Pr\{\omega = \phi^{-1}(c)\} > 0 \\ 0, & \Pr\{\omega = \phi^{-1}(c)\} = 0, \end{cases}$$

and construct that

$$\partial_x\varphi(x^*,\omega) \ni g_1(\omega) = \begin{cases} \begin{bmatrix}a^{\mathrm{d}} & p^{\mathrm{d}}\end{bmatrix}^\top, & \omega - q^{\mathrm{s},*} < 0 \leq q^{\mathrm{d},*} \text{ (Case 1)} \\ \begin{bmatrix}a^{\mathrm{d}} & r^{\mathrm{d}}\end{bmatrix}^\top + k\begin{bmatrix}0 & p^{\mathrm{d}} - r^{\mathrm{d}}\end{bmatrix}^\top, & 0 = \omega - q^{\mathrm{s},*} < q^{\mathrm{d},*} \text{ (Case 5)} \\ \begin{bmatrix}a^{\mathrm{d}} & r^{\mathrm{d}}\end{bmatrix}^\top, & 0 < \omega - q^{\mathrm{s},*} \leq q^{\mathrm{d},*} \text{ (Case 2, 6)} \\ \begin{bmatrix}a^{\mathrm{d}} - r^{\mathrm{d}} & 0\end{bmatrix}^\top, & 0 < q^{\mathrm{d},*} < \omega - q^{\mathrm{s},*} \text{ (Case 3).} \end{cases}$$

We then have

$$
\begin{aligned}
&\partial_x \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(x^*,\omega)\right] \;\ni\; \mathbb{E}^{\omega\sim\mathcal{F}}\left[g_1(\omega)\right] \\
=&\begin{bmatrix} a^{\mathrm{d}} \\ 0 \end{bmatrix} + \left(\phi(\phi^{-1}(c)) - \Pr\{\omega=\phi^{-1}(c)\}\right)\begin{bmatrix} 0 \\ p^{\mathrm{d}} \end{bmatrix} + \left(\phi(D) - \phi(\phi^{-1}(c)) - \Pr\{\omega = D\}\right)\begin{bmatrix} 0 \\ r^{\mathrm{d}} \end{bmatrix} \\
&+ (1-\phi(D))\begin{bmatrix} -r^{\mathrm{d}} \\ 0 \end{bmatrix} + \left(c - \phi(\phi^{-1}(c)) + \Pr\{\omega=\phi^{-1}(c)\}\right)\begin{bmatrix} 0 \\ p^{\mathrm{d}} - r^{\mathrm{d}} \end{bmatrix} \\
&+ \Pr\{\omega=\phi^{-1}(c)\}\begin{bmatrix} 0 \\ r^{\mathrm{d}} \end{bmatrix} + \Pr\{\omega = D\}\begin{bmatrix} 0 \\ r^{\mathrm{d}} \end{bmatrix} \\
=&\left(a^{\mathrm{d}} - r^{\mathrm{d}}(1-\phi(D))\right)\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda^{\mathrm{DA},*}\begin{bmatrix} 1 \\ 1 \end{bmatrix}.
\end{aligned}
$$

Similarly, we can also prove that $\lambda^{\mathrm{DA},*}\begin{bmatrix} 1 & 1 \end{bmatrix}^\top \in \partial_x \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(x^*,\omega)\right]$ if $\phi^{-1}(c) = D$. Since $\lambda^{\mathrm{DA},*}\begin{bmatrix} -1 & -1 \end{bmatrix}^\top \in N_S(x^*)$, it holds that

$$
0 = \lambda^{\mathrm{DA},*}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \lambda^{\mathrm{DA},*}\begin{bmatrix} -1 \\ -1 \end{bmatrix} \;\in\; \partial_x \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(x^*,\omega)\right] + N_S(x^*), \tag{3.8}
$$

which shows the optimality of the candidate solution.

- **Step 3.2: Examining the Optimality Conditions ($\boldsymbol{\phi(D) \leq c}$)**

For $\phi(D) \leq c$, the candidate solution is $q^{\mathrm{s},*} = D, q^{\mathrm{d},*} = 0, \lambda^{\mathrm{DA},*} = p^{\mathrm{d}}\phi(D)$. Case 1, 7, 4 in Table 3.1 could happen with the probability of $\phi(D) - \Pr\{\omega = D\}$, $\Pr\{\omega = D\}$, and $1 - \phi(D)$, respectively. It is trivial that

$$
\partial_x \varphi(x^*,\omega) \;\ni\; g_2(\omega) = \begin{cases} \begin{bmatrix} a^{\mathrm{d}} & p^{\mathrm{d}} \end{bmatrix}^\top, & \omega - q^{\mathrm{s},*} \leq 0 = q^{\mathrm{d},*} \text{ (Case 1, 7)} \\ \begin{bmatrix} a^{\mathrm{d}} - r^{\mathrm{d}} & 0 \end{bmatrix}^\top, & 0 = q^{\mathrm{d},*} < \omega - q^{\mathrm{s},*} \text{ (Case 4).} \end{cases}
$$

And we have

$$
\begin{aligned}
&\partial_x \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(x^*,\omega)\right] \;\ni\; \mathbb{E}^{\omega\sim\mathcal{F}}\left[g_2(\omega)\right] \\
=&\begin{bmatrix} a^{\mathrm{d}} \\ 0 \end{bmatrix} + (\phi(D) - \Pr\{\omega = D\})\begin{bmatrix} 0 \\ p^{\mathrm{d}} \end{bmatrix} + \Pr\{\omega = D\}\begin{bmatrix} 0 \\ p^{\mathrm{d}} \end{bmatrix} + (1-\phi(D))\begin{bmatrix} -r^{\mathrm{d}} \\ 0 \end{bmatrix} \\
=&\begin{bmatrix} a^{\mathrm{d}} - r^{\mathrm{d}}(1-\phi(D)) \\ p^{\mathrm{d}}\phi(D) \end{bmatrix} = \lambda^{\mathrm{DA},*}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \lambda^{\mathrm{d},*}\begin{bmatrix} 1 \\ 0 \end{bmatrix},
\end{aligned}
$$

where $\lambda^{\mathrm{d},*} = a^{\mathrm{d}} - r^{\mathrm{d}} - (p^{\mathrm{d}} - r^{\mathrm{d}})\phi(D) \geq 0$. As $\lambda^{\mathrm{DA},*}\begin{bmatrix} -1 \\ -1 \end{bmatrix} + \lambda^{\mathrm{d},*}\begin{bmatrix} -1 \\ 0 \end{bmatrix} \in N_S(x^*)$, we have

$$
0 = (\lambda^{\mathrm{DA},*}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \lambda^{\mathrm{d},*}\begin{bmatrix} 1 \\ 0 \end{bmatrix}) + (\lambda^{\mathrm{DA},*}\begin{bmatrix} -1 \\ -1 \end{bmatrix} + \lambda^{\mathrm{d},*}\begin{bmatrix} -1 \\ 0 \end{bmatrix}) \;\in\; \partial_x \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(x^*,\omega)\right] + N_S(x^*), \tag{3.9}
$$

proving the optimality of the candidate solution.

In conclusion, equation (3.8) and (3.9) together prove that (3.5) gives the optimal solution to the electricity market. □

**Remark 1.** *If the production distribution $\mathcal{F}$ is discrete, then the stochastic electricity market model is equivalent to the scenario-based stochastic market models studied in [4, 6–10].*

## 3.2 Insights of the Solution

### 3.2.1 Renewable Penetration and Optimal Dispatch

Figure 3.1 illustrates the sketch of the stochastic DA market solutions regarding different renewable penetrations.

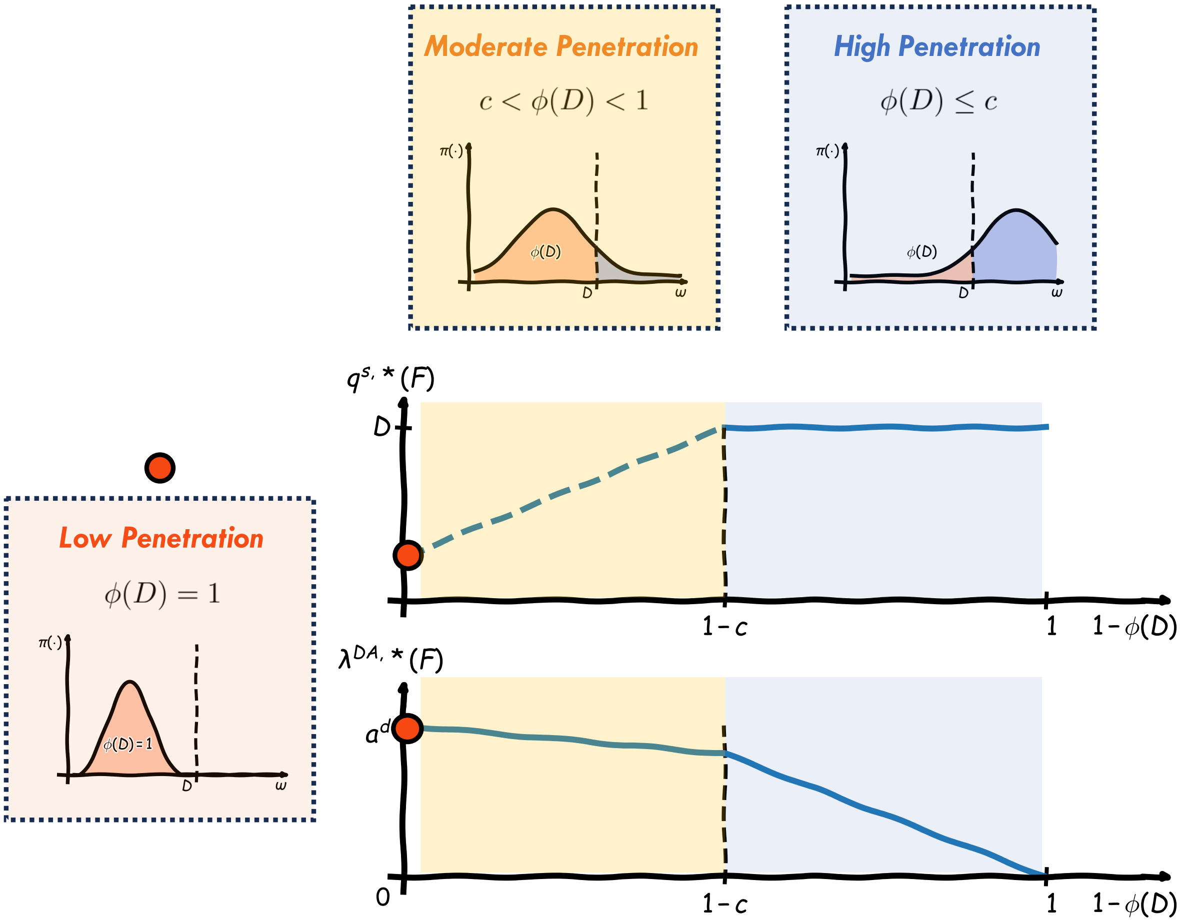



Figure 3.1: Sketch of the stochastic DA market solution ($1 - \phi(D) = \Pr\{\omega > D\}$)

The value $\phi(D)$ plays a key role in determining the optimal dispatches and the clearing price. It represents the probability of the WPP producing less than the demand as $\phi(D) = \Pr\{\omega \leq D\}$. Conversely, the probability that the WPP produces more than the demand is $1-\phi(D) = \Pr\{\omega > D\}$. When $\omega > D$ occurs, the market is flooded with cheap renewable energy, and the TPP is not required to generate any electricity. Thus, $\phi(D)$ measures the WPP's producing capability and serves as an indicator of renewable penetration in the market.

- **Low renewable penetration ($\phi(D) = 1$)**: Under low renewable penetration, the WPP has no chance to meet all the demands, necessitating the involvement of the TPP to supply the remaining energy.
- **Moderate renewable penetration ($c < \phi(D) < 1$)**: Mathematically, the constraint $q^{\mathrm{s}} \leq D$ is non-binding and the optimal WPP dispatch $q^{\mathrm{s},*}(\mathcal{F})$ is less than the demand $D$. The MO believes that the likelihood of the WPP meeting the entire demand is not high enough and merely assigns part of it to the WPP.
- **High renewable penetration ($\phi(D) \leq c$)**: When the market is characterized by *high*

*renewable penetration*, the MO has confidence in the WPP's ability to meet the demand and assigns the entire demand to the WPP as $q^{\mathrm{s},*}(\mathcal{F}) = D$.

We further discuss the optimal WPP dispatch $q^{\mathrm{s},*} = \phi^{-1}(c) \in [0, D]$ under low/moderate renewable penetration. It is equivalent to the condition $\phi(q^{\mathrm{s},*}) = c$. The value $\phi(q^{\mathrm{s},*}) := \Pr\{\omega \leq q^{\mathrm{s},*}\}$ represents WPP's shortfall likelihood. More specifically, as illustrated in Figure 3.2, the optimal WPP dispatch balances the risk of shortfall and surplus such that the shortfall likelihood equals the threshold $c$, which is determined by incremental bid prices. Note that both WPP shortfall and surplus are undesirable (see 3.3 Illustrative Examples for more details). We thus call $c$ the *optimal shortfall likelihood*.

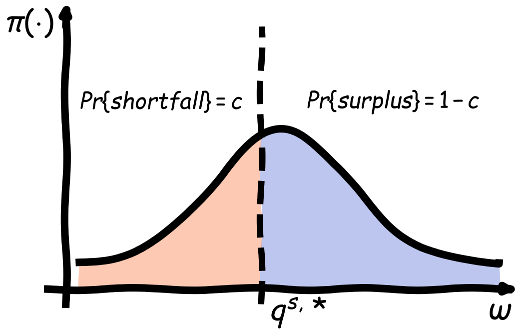



Figure 3.2: Relationship between likelihood of WPP shortfall/surplus and optimal WPP dispatch

### 3.2.2 Interpretation of the Pricing Rule

The measure of renewable penetration $\phi(D)$ also influences the market price. In the case of low renewable penetration, where the WPP is unlikely to meet the demand by itself, the market price $\lambda^{\mathrm{DA},*}(\mathcal{F})$ equals the TPP's DA marginal production cost $a^{\mathrm{d}}$. This situation corresponds to the deterministic market case depicted in Figure 2.1a where the WPP produces less than the demand. On the other hand, if the WPP production is guaranteed to exceed the demand, i.e., $\phi(D) = 0$, the price drops to $\lambda^{\mathrm{DA},*}(\mathcal{F}) = 0$, reflecting the deterministic market case shown in Figure 2.1b. As renewable penetration increases between these two extremes, the market price decreases accordingly.

We develop further interpretation of the pricing rule using the Lagrangian multiplier $\lambda^{\mathrm{DA},*}$. To start with, note that the first-order derivative of the expected total social cost to the decision variables ($q^{\mathrm{d}}$ and $q^{\mathrm{s}}$) measures the *total marginal cost*:

$$\begin{aligned}
\underbrace{\nabla_{q^{\mathrm{d}}} \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]}_{\text{TPP total marginal cost}} &= \underbrace{a^{\mathrm{d}}}_{\text{TPP immediate marginal cost}} + \underbrace{\nabla_{q^{\mathrm{d}}} \mathbb{E}^{\omega\sim\mathcal{F}}\left[C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]}_{\text{TPP future marginal cost}} \\
&= a^{\mathrm{d}} + r^{\mathrm{d}}\left(\phi(q^{\mathrm{d}} + q^{\mathrm{s}}) - 1\right) \\
\underbrace{\nabla_{q^{\mathrm{s}}} \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]}_{\text{WPP total marginal cost}} &= \underbrace{0}_{\text{WPP immediate marginal cost}} + \underbrace{\nabla_{q^{\mathrm{s}}} \mathbb{E}^{\omega\sim\mathcal{F}}\left[C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]}_{\text{WPP future marginal cost}} \\
&= (p^{\mathrm{d}} - r^{\mathrm{d}})\phi(q^{\mathrm{s}}) + r^{\mathrm{d}}\phi(q^{\mathrm{d}} + q^{\mathrm{s}}).
\end{aligned}$$

This indicates that the marginal production of a producer in the stochastic DA market not only incurs a DA marginal production cost, but also causes a marginal outcome in the RT market. Hence, the former is referred to as the *immediate marginal cost*, while the latter is referred to as the *future marginal cost*.

Recall from the KKT optimality conditions in (3.7), at the optimal solution, it holds that

$$\begin{aligned}
&\nabla_{q^{\mathrm{d}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d},*},q^{\mathrm{s},*},\omega)\right]-\lambda^{\mathrm{DA},*}-\lambda^{\mathrm{d},*}=0,\\
&\nabla_{q^{\mathrm{s}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d},*},q^{\mathrm{s},*},\omega)\right]-\lambda^{\mathrm{DA},*}-\lambda^{\mathrm{s},*}=0,\\
&0\leq q^{\mathrm{d},*}\perp\lambda^{\mathrm{d},*}\geq 0,\qquad 0\leq q^{\mathrm{s},*}\perp\lambda^{\mathrm{s},*}\geq 0.
\end{aligned}$$

These conditions ensure that when a producer is actively producing energy in the market (with positive production), $\lambda^{\mathrm{DA},*}$ is equal to its total marginal cost. Take the TPP as an example, if it is actively producing, i.e., $q^{\mathrm{d}}>0$, we have $\lambda^{\mathrm{d},*}=0$ due to complementary slackness and hence,

$$\nabla_{q^{\mathrm{d}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d},*},q^{\mathrm{s},*},\omega)\right]-\lambda^{\mathrm{DA},*}=0.$$

Similar results apply to the WPP. Therefore, the Lagrangian multiplier $\lambda^{\mathrm{DA},*}$ matches the **total marginal cost of active producers**. Under low or moderate renewable penetration, where both producers are actively producing energy, the market clearing price $\lambda^{\mathrm{DA},*}(\mathcal{F})$ matches their total marginal costs. Conversely, when the market is under high renewable penetration, the TPP stops producing energy and the WPP's total marginal cost determines the market price. The total marginal cost of the TPP and the WPP react differently as the degree of renewable penetration changes, which explains the piecewise linear shape of the pricing rule described in (3.5c), with $\phi(D)=c$ being the breakpoint.

In fact, the MO of the stochastic electricity market is idealistic, since it assumes all producers are with foresight (evaluate their marginal costs using the total marginal cost). If they are, they will agree on the optimal dispatches determined by the MO after it sets $\lambda^{\mathrm{DA},*}$ as the market price. However, myopic producers, who only characterize their marginal costs by the DA marginal production costs (immediate marginal costs) without considering the future outcomes, would not be happy about the MO's dispatch decisions.

This result is fundamentally different from the traditional deterministic DA market (2.1), where the market clearing price matches the **highest DA marginal production cost among active producers**. The deterministic DA market assumes all producers to be myopic.

**Remark 2.** *The dual variable as the clearing price is elegant but very fragile, as a different but primally equivalent formulation could change the physical meaning of the dual variable.*

## 3.3 Illustrative Examples

### 3.3.1 Shape of the Social Production Cost Function

Let the inelastic demand $D$ equal 100 MW. And let the WPP production distribution be $\mathcal{F}=U(30,130)$ MW. The DA marginal production cost of the TPP is $a^{\mathrm{d}}=10$ \$/MWh. The TPP has some flexible units capable of producing energy on short notice, yet at a higher marginal production cost of $p^{\mathrm{d}}=11$ \$/MWh. And the TPP is willing to buy cheaper energy in the RT market at $r^{\mathrm{d}}=9$ \$/MWh.

Recall that $\varphi(q^{\mathrm{d}},q^{\mathrm{s}},\omega)$ represents the total production cost upon a realization of the actual WPP production $\omega$, and the DA dispatches $q^{\mathrm{d}}$ and $q^{\mathrm{s}}$. Suppose the actual WPP's production is $\omega=30$ MW. Note that since $q^{\mathrm{d}}+q^{\mathrm{s}}=D$, this function is only with one degree of freedom. Figure 3.3 illustrates the shape of $\varphi(D-q^{\mathrm{s}},q^{\mathrm{s}},30)$. The optimal DA decision is $q^{\mathrm{s},*}=30$ MW and $q^{\mathrm{d},*}=70$ MW. This optimal WPP dispatch matches the actual production. It gives rise to the optimal cost $70a^{\mathrm{d}}=700$\$. If the MO decides $q^{\mathrm{s}}=40$ MW and $q^{\mathrm{d}}=60$ MW, the actual production creates a wind power shortfall of 10 MW. To compensate for this shortfall,

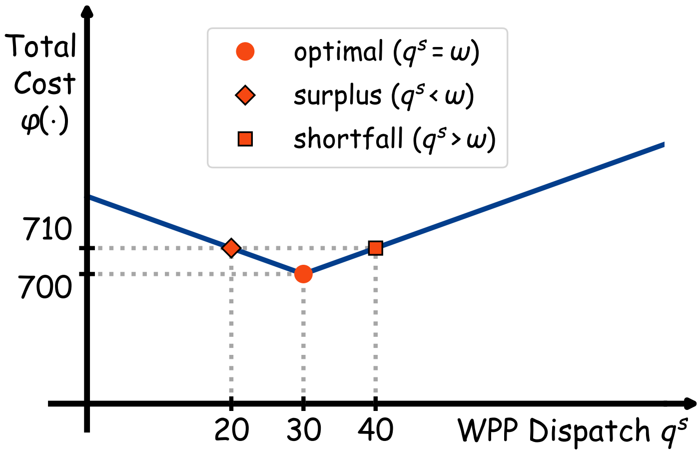



Figure 3.3: Total production cost versus DA WPP dispatch ($\omega = 30$ MW)

the TPP needs to produce an additional 10 MW at the cost of $p^{\mathrm{d}}$. The social cost then becomes $60a^{\mathrm{d}} + 10p^{\mathrm{d}} = 710\$$, which increases $10(p^{\mathrm{d}} - a^{\mathrm{d}}) = 10\Delta a^{\mathrm{d},+} = 10\$$.

On the other hand, if the MO assigns too little energy to the WPP in the DA market, the cost also increases. For example, with $q^{\mathrm{s}} = 20$ MW and $q^{\mathrm{d}} = 80$ MW, the WPP produces 10 MW more than expected, the TPP can buy this excess energy on the RT market at $r^{\mathrm{d}} = 9$ \$/MWh (according to equation (2.5)) which is cheaper than producing it. This results in a social cost of $80a^{\mathrm{d}} - 10r^{\mathrm{d}} = 710\$$. The additional cost, $10(a^{\mathrm{d}} - r^{\mathrm{d}}) = 10\Delta a^{\mathrm{d},-} = 10\$$, can be interpreted as a consequence of unrecoverable cost for overly scheduled resources on the DA market. For example, the TPP schedules 8 workers to produce 80 MW while only 7 workers are needed to produce 70 MW in total.

In general, the optimal DA WPP dispatch equals the actual production, *if only such perfect information were known in advance*. Any deviation results in shortfalls or surpluses, incurring extra social costs. Specifically, as the TPP needs to produce extra energy at the marginal cost of $p^{\mathrm{d}}$ for WPP shortfall, $\Delta a^{\mathrm{d},+}$ can be viewed as the net penalty per unit of WPP shortfall, while $\Delta a^{\mathrm{d},-}$ is the net penalty per unit of WPP surplus. This described solution corresponds to the *wait-and-see* (WS) solution [17]. It is trivial that if the MO knows the accurate WPP production $\omega$, at the DA stage, the optimal decision is $q^{\mathrm{s},*} = \min(\omega, D)$, such that it incurs neither WPP shortfall nor surplus and no trading is required in the RT market. Thus, the expected social cost is $\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] = \mathbb{E}^{\omega\sim\mathcal{F}}\left[a^{\mathrm{d}}(D - \min(\omega, D))\right] = 245.0\$$.

### 3.3.2 Lower Expected Total Social Production Cost

The WS solution achieves the lowest social cost, by avoiding shortfall/surplus in any scenario of WPP production. However, it assumes perfect information and is thus impractical. We thus compare the outcomes of the stochastic DA market (3.2) and the deterministic DA market (2.1) under imperfect information.

We first analyse the outcomes of the stochastic DA market. According to the optimal dispatch solution in equation (3.5a), for the case $a^{\mathrm{d}} = 10$ \$/MWh, $p^{\mathrm{d}} = 11$ \$/MWh, $r^{\mathrm{d}} = 9$ \$/MWh, the upward and downward incremental bid prices are symmetric: $\Delta a^{\mathrm{d},+} = \Delta a^{\mathrm{d},-} = 1$ \$/MWh. This results in an optimal shortfall likelihood of $c = \frac{\Delta a^{\mathrm{d},-}}{\Delta a^{\mathrm{d},+} + \Delta a^{\mathrm{d},-}} = 0.5$, and the optimal WPP dispatch $q^{\mathrm{s},*} = \phi^{-1}(\frac{1}{2}) = 30 + 100 \times \frac{1}{2} = 80$ MW, aligning with the mean production.

However, if the flexible units of the TPP exhibit a higher marginal production cost, e.g.,

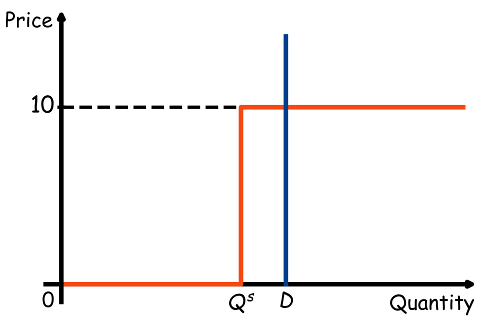
$p^{\mathrm{d}} = 14$ \$/MWh, the net penalty for WPP shortfall becomes $\Delta a^{\mathrm{d},+} = 4$, exceeding the net penalty for WPP surplus. As a response, to reduce the risk of WPP shortfall, optimal shortfall likelihood drops to $c = \frac{1}{5}$ and the optimal dispatch decreases to $q^{\mathrm{s},*} = \phi^{-1}(\frac{1}{5}) = 30 + 100 \times \frac{1}{5} = 50$ MW. Conversely, if the TPP gets some flexible units with a lower marginal production cost of $p^{\mathrm{d}} = 10.25$ \$/MWh, the net penalty for WPP shortfall becomes lower than the surplus penalty. As a shortfall is less costly than a surplus, the optimal shortfall likelihood increases to $c = \frac{4}{5}$ and $\phi(D) = 0.7 < c$, characterizing a high renewable penetration. The MO assigns $q^{\mathrm{s},*} = D = 100$ MW. In conclusion, when the upward and downward incremental bid prices are not symmetric, i.e., $\Delta a^{\mathrm{d},+} \neq \Delta a^{\mathrm{d},-}$, either surplus or shortfall will be less preferred and the optimal dispatch deviates from the mean $\mu$.

Table 3.2 lists the optimal WPP dispatch and the optimal expected social cost for each case. As a comparison, we analyze the deterministic market formulation (2.1), which requires a forecast of the WPP production $Q^{\mathrm{s}}$. It is reasonable to use the mean value of the distribution as the forecast, i.e., $Q^{\mathrm{s}} = 80$ MW. Solving problem (2.1) with such parameters, we obtain $q^{\mathrm{s},*} = Q^{\mathrm{s}} = 80$ MW. The MO will utilize all the energy it expects the WPP to produce. Notably, since the RT market cost is not considered in this formulation, this dispatch remains the same regardless of the incremental bid prices. This decision exhibits higher expected social costs than the stochastic formulation in the cases of insymmetric incremental bid prices.

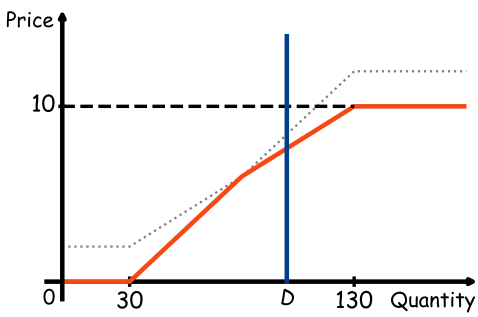
Table 3.2: WPP dispatches, market prices, and expected total social production costs of the Stochastic and Deterministic Markets ($\mathcal{F} = U(30, 130)$ MW)

| Inc. Bid Prices | Stochastic | | | Deterministic | | |
|---|---|---|---|---|---|---|
| | $q^{\mathrm{s},*}$ (MW) | Price (\$/MWh) | Exp. Prod. Cost (\$) | $q^{\mathrm{s},*}$ (MW) | Price (\$/MWh) | Exp. Prod. Cost (\$) |
| $\Delta a^{\mathrm{d},+} = \Delta a^{\mathrm{d},-} = 1$ | 80 | 7.3 | 265.5 | 80 | 10 | 265.5 |
| $\Delta a^{\mathrm{d},+} = 4, \Delta a^{\mathrm{d},-} = 1$ | 50 | 7.3 | 280.5 | 80 | 10 | 303.0 |
| $\Delta a^{\mathrm{d},+} = 0.25, \Delta a^{\mathrm{d},-} = 1$ | 100 | 7.125 | 251.125 | 80 | 10 | 256.125 |

When we take a further look at the supply curves of both the deterministic market and the stochastic market as shown in Figure 3.4, it is obvious that the market price in the deterministic setting is “binary”. More specifically, when the producers are bidding with constant prices, the market price is either the marginal production cost of the TPP ($a^{\mathrm{d}}$ \$/MWh), or the marginal production cost of the WPP (0 \$/MWh). In the stochastic setting, on the other hand, due to the additional second-stage cost, the supply curve becomes *continuous*, making the market price less sensitive to the relationship between the WPP production and the demand quantity, which contributes to more stable profits for the producers.




(a) Deterministic




(b) Stochastic

Figure 3.4: The demand curve (blue) and the supply curve (red) for both market schemes

# Chapter 4

# Incentive Compatibility

Previously, we have assumed zero WPP marginal production costs as in Assumption 2. Under the condition that the MO knows the WPP production distribution, the WPP only acts as a price-taker, which might not be the role it would like to play as the renewable penetration increases. Hence, in this chapter, we introduce the WPP as the *distribution bidder* to allow it to actively influence the market. More specifically, while we assume zero WPP marginal production costs (therefore it does not bid for prices), we assign the WPP with the power of providing its own production distribution to the MO. We further assume that the WPP has the perfect information of its real distribution, denoted by $\tilde{\mathcal{F}}$, while it is allowed to bid with a distribution $\mathcal{F}$, which might or might not align with the true distribution. The TPP, on the other hand, still bids for prices and thus is referred to as the *price bidder*. As before, we still assume truthful bids and nonnegative incremental bid prices of the TPP as in Assumption 1 and 3.

In this chapter, we analyze the incentive compatibility of both the price bidder and the distribution bidder. We first assume a truthful distribution bid and study cost recovery of the two producers to see if they have the incentive to participate in the market. And then we focus on whether the distribution bidder will bid truthfully or not.

## 4.1 Cost Recovery

### 4.1.1 Formulations

In order to study cost recovery, We first formalize the producers' profits in the stochastic electricity market (3.2).

Both producers are paid or paying upon the clearing of the DA market and the RT market separately. We can thus discuss the profits of these two stages separately.

The outcomes of the DA market is deterministic, since its functionality is more about scheduling. With the WPP submitting a distribution bid $\mathcal{F}$, the MO computes the optimal dispatches $q^{\mathrm{d},*}(\mathcal{F}), q^{\mathrm{s},*}(\mathcal{F})$ and the market price $\lambda^{\mathrm{DA},*}(\mathcal{F})$. For simplification, within this section, we omit the apprentices and use $q^{\mathrm{d},*}$, $q^{\mathrm{s},*}$, $\lambda^{\mathrm{DA},*}$ to represent them. Since the TPP and the WPP produce each unit of energy at a cost of $a^{\mathrm{d}}$ and 0, their DA profits are

$$\begin{aligned} \text{TPP DA Profit} \quad & P_1^{\mathrm{d}}(\mathcal{F}) = \left(\lambda^{\mathrm{DA},*} - a^{\mathrm{d}}\right) q^{\mathrm{d},*} \\ \text{WPP DA Profit} \quad & P_1^{\mathrm{s}}(\mathcal{F}) = \lambda^{\mathrm{DA},*} q^{\mathrm{s},*}. \end{aligned}$$

Note that the profits are functions of the bidding distribution $\mathcal{F}$. The TPP's profit is nonpositive since we always have $\lambda^{\mathrm{DA},*}(\mathcal{F}) \leq a^{\mathrm{d}}$ according to the optimal pricing solution in (3.5c). The

TPP is thus not profitable in the DA market.

Their profits in the RT market, on the other hand, exhibit stochasticity when analyzing in advance. The producers' actions (selling or buying energy) and the RT market price (recall from Equation (2.5)) depend on the realization of WPP's production $\omega$.

- $\mathbf{0 \leq \omega \leq q^{s,*}(\mathcal{F})}$: The WPP produces a shortfall and has to buy extra energy from the TPP at the price $p^{\mathrm{d}}$, which is the RT marginal production cost of the TPP. Hence, the WPP loses a profit of $p^{\mathrm{d}}\left(q^{\mathrm{s},*}(\mathcal{F})-\omega\right)$ and the TPP earns 0.
- $\mathbf{q^{s,*}(\mathcal{F}) < \omega \leq D}$: The WPP can sell the extra production to the TPP at a price of $r^{\mathrm{d}}$. $r^{\mathrm{d}}$ represents the highest price that the TPP is willing to pay such that it does not exhibit a financial loss after reducing its scheduled production. Consequently, while the WPP earns $r^{\mathrm{d}}\left(\omega-q^{\mathrm{s},*}(\mathcal{F})\right)$, the TPP still makes no profits.
- $\mathbf{\omega > D}$: The RT market price is 0, leading to zero profit of the WPP. The TPP can simply stop producing any energy and save part of the production cost $r^{\mathrm{d}}q^{\mathrm{d},*}(\mathcal{F})$.

Considering these three cases, we are then able to formulate the RT profits for both producers. While the distribution bid $\mathcal{F}$ determines the dispatches and the market price, it is the true WPP production distribution $\tilde{\mathcal{F}}$ that counts for the true expected RT profits.

$$\begin{aligned}
\text{TPP RT Profit}\quad P_2^{\mathrm{d}}(\mathcal{F}) &= \int_D^{+\infty} r^{\mathrm{d}}q^{\mathrm{d},*}\tilde{\pi}(\omega)\mathrm{d}\omega = r^{\mathrm{d}}q^{\mathrm{d},*}\left(1-\tilde{\phi}(D)\right)\\
\text{WPP RT Profit}\quad P_2^{\mathrm{s}}(\mathcal{F}) &= -\int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}}(q^{\mathrm{s},*}-\omega)\tilde{\pi}(\omega)\mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}}(\omega-q^{\mathrm{s},*})\tilde{\pi}(\omega)\mathrm{d}\omega
\end{aligned}$$

Adding the expected profits from these two stages together, we have the total profits for both producers:

$$\text{TPP Profit}\quad P^{\mathrm{d}}(\mathcal{F}) = \left(\lambda^{\mathrm{DA},*}-a^{\mathrm{d}}\right)q^{\mathrm{d},*} + r^{\mathrm{d}}q^{\mathrm{d},*}\left(1-\tilde{\phi}(D)\right) \tag{4.1a}$$

$$\text{WPP Profit}\quad P^{\mathrm{s}}(\mathcal{F}) = \lambda^{\mathrm{DA},*}q^{\mathrm{s},*} - \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}}(q^{\mathrm{s},*}-\omega)\tilde{\pi}(\omega)\mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}}(\omega-q^{\mathrm{s},*})\tilde{\pi}(\omega)\mathrm{d}\omega \tag{4.1b}$$

We can further develop a link between the WPP's RT profit and the expected total social cost. To do so, we differentiate two different costs: 1. the MO's cost $\mathbb{E}^{\boldsymbol{\omega}\sim\boldsymbol{\mathcal{F}}}\left[\varphi(q^{\mathrm{d}},q^{\mathrm{s}},\omega)\right]$, and 2. the true cost $\mathbb{E}^{\boldsymbol{\omega}\sim\tilde{\boldsymbol{\mathcal{F}}}}\left[\varphi(q^{\mathrm{d}},q^{\mathrm{s}},\omega)\right]$. Their only difference is the distribution that $\omega$ follows. With the MO's decisions $q^{\mathrm{d},*}(\mathcal{F})$ and $q^{\mathrm{s},*}(\mathcal{F})$, the *true expected total social cost* is

$$\begin{aligned}
\tilde{C}(\mathcal{F}) &= \mathbb{E}^{\omega\sim\tilde{\mathcal{F}}}\left[\varphi(q^{\mathrm{d},*}(\mathcal{F}),q^{\mathrm{s},*}(\mathcal{F}),\omega)\right]\\
&= a^{\mathrm{d}}q^{\mathrm{d},*} + \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}}(q^{\mathrm{s},*}-\omega)\tilde{\pi}(\omega)\mathrm{d}\omega - \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}}(\omega-q^{\mathrm{s},*})\tilde{\pi}(\omega)\mathrm{d}\omega - r^{\mathrm{d}}q^{\mathrm{d},*}\left(1-\tilde{\phi}(D)\right),
\end{aligned}$$

as a function of the distribution bid $\mathcal{F}$. It is clear that the WPP's RT profit is related to the true expected total social cost:

$$P_2^{\mathrm{s}}(\mathcal{F}) = a^{\mathrm{d}}q^{\mathrm{d},*} - r^{\mathrm{d}}q^{\mathrm{d},*}\left(1-\tilde{\phi}(D)\right) - \tilde{C}(\mathcal{F}).$$

If the distribution bid $\mathcal{F}$ announces a low/moderate renewable penetration such that $\phi(D) > c$, according to the closed-form solution to the DA market in (3.5), the market price is $\lambda^{\mathrm{DA},*} =$

$a^{\mathrm{d}} - r^{\mathrm{d}}(1 - \phi(D))$. We then have

$$P^{\mathrm{d}}(\mathcal{F}) = r^{\mathrm{d}} q^{\mathrm{d},*} \left( \phi(D) - \tilde{\phi}(D) \right) \tag{4.2a}$$

$$P^{\mathrm{s}}(\mathcal{F}) = a^{\mathrm{d}} D + r^{\mathrm{d}} D \left( \tilde{\phi}(D) - 1 \right) + r^{\mathrm{d}} q^{\mathrm{s},*} \left( \phi(D) - \tilde{\phi}(D) \right) - \tilde{C}(\mathcal{F}). \tag{4.2b}$$

Otherwise, if $\mathcal{F}$ exhibits high renewable penetration with $\phi(D) \leq c$, the optimal dispatches are $q^{\mathrm{d},*} = 0$, $q^{\mathrm{s},*} = D$, and the market price is $\lambda^{\mathrm{DA},*} = p^{\mathrm{d}} \phi(D)$. In this case, we have

$$P^{\mathrm{d}}(\mathcal{F}) = 0, \quad P^{\mathrm{s}}(\mathcal{F}) = p^{\mathrm{d}} D \phi(D) - \tilde{C}(\mathcal{F}). \tag{4.3}$$

**Theorem 4.1.1.** *If the WPP bids truthfully (i.e., $\mathcal{F} = \tilde{\mathcal{F}}$), then the TPP makes zero expected total profit, while the WPP itself makes nonnegative expected profit.*

*Proof.* When the WPP submits a truthful bid, it holds that $\phi(D) = \tilde{\phi}(D)$. According to Equation (4.2a), we have $P^{\mathrm{d}}(\tilde{\mathcal{F}}) = r^{\mathrm{d}} q^{\mathrm{d},*} \left( \tilde{\phi}(D) - \tilde{\phi}(D) \right) = 0$ when $\tilde{\mathcal{F}}$ exhibits a low/moderate renewable penetration. And since the TPP does not participate in the market at all under a high renewable penetration, the TPP always makes zero expected total profit.

We then focus on the WPP's expected total profit. From (4.1b), we have

$$\begin{aligned} P^{\mathrm{s}}(\tilde{\mathcal{F}}) &= \lambda^{\mathrm{DA},*} q^{\mathrm{s},*} - \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}} (q^{\mathrm{s},*} - \omega) \tilde{\pi}(\omega) \mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}} (\omega - q^{\mathrm{s},*}) \tilde{\pi}(\omega) \mathrm{d}\omega \\ &= \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + \lambda^{\mathrm{DA},*} q^{\mathrm{s},*} - p^{\mathrm{d}} q^{\mathrm{s},*} \tilde{\phi}(q^{\mathrm{s},*}) - r^{\mathrm{d}} q^{\mathrm{s},*} \left( \tilde{\phi}(D) - \tilde{\phi}(q^{\mathrm{s},*}) \right) \end{aligned}$$

For a low/moderate renewable penetration case, we have $\lambda^{\mathrm{DA},*} = a^{\mathrm{d}} - r^{\mathrm{d}} \left( 1 - \tilde{\phi}(D) \right)$ and $q^{\mathrm{s},*} = \tilde{\phi}^{-1}(c)$. Hence,

$$\begin{aligned} P^{\mathrm{s}}(\tilde{\mathcal{F}}) &= \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + (a^{\mathrm{d}} - r^{\mathrm{d}}) q^{\mathrm{s},*} - (p^{\mathrm{d}} - r^{\mathrm{d}}) q^{\mathrm{s},*} \tilde{\phi}(q^{\mathrm{s},*}) \\ &= \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + (a^{\mathrm{d}} - r^{\mathrm{d}}) q^{\mathrm{s},*} - (p^{\mathrm{d}} - r^{\mathrm{d}}) q^{\mathrm{s},*} c \\ &= \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega > 0. \end{aligned}$$

And when $\tilde{\mathcal{F}}$ exhibits a high renewable penetration, we have $\lambda^{\mathrm{DA},*} = p^{\mathrm{d}} \tilde{\phi}(D)$ and $q^{\mathrm{s},*} = D$. Moreover, $\tilde{\pi}(\cdot)$ has no density in the range of $[0, D]$. Thus, it holds that

$$\begin{aligned} P^{\mathrm{s}}(\tilde{\mathcal{F}}) &= \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + p^{\mathrm{d}} D \tilde{\phi}(D) - p^{\mathrm{d}} D \tilde{\phi}(D) + r^{\mathrm{d}} D \left( \tilde{\phi}(D) - \tilde{\phi}(D) \right) \\ &= 0. \end{aligned}$$

Therefore, under both cases, the WPP's expected total profit is

$$P^{\mathrm{s}}(\tilde{\mathcal{F}}) = \int_0^{q^{\mathrm{s},*}} p^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega + \int_{q^{\mathrm{s},*}}^{D} r^{\mathrm{d}} \omega \tilde{\pi}(\omega) \mathrm{d}\omega \geq 0.$$

In conclusion, when the WPP bids truthfully, both producers' expected total profits are nonnegative. Notably, when the true distribution $\tilde{\mathcal{F}}$ exhibits a high renewable penetration, both producers make zero expected total profits. □

Theorem 4.1.1 indicates that both producers achieve cost recovery in expectation and thus have the incentive to participate if they are risk-neutral.

### 4.1.2 Example

While the stochastic electricity market ensures cost recovery in expectation for both producers, the deterministic market does not. To compare the producers' outcomes in these two markets, we revisit the market example in Chapter 3.3, where the WPP's true production follows the probability distribution $\tilde{\mathcal{F}} = U(30, 130)$ MW

- **Example 1 ($\omega \sim U(30, 130)$ MW)**

Table 4.1 reports the expected profits for both the WPP and TPP in the stochastic and deterministic markets together with the consumer's costs. Since the market clearing price in the stochastic market is generally lower, both producers earn less profit compared to the deterministic market. In particular, the TPP makes zero profit in the stochastic market when it submits constant bid prices. Naturally, producers would favor the deterministic market due to the higher profits. However, consumers benefit more from the stochastic market because of the lower market price. Overall, as previously discussed, when considering the expected social welfare (represented by the expected total social cost) the stochastic market is a more efficient model.

- **Example 2 ($\omega \sim U(80, 130)$ MW)**

It is not always true that the producers earn more profit in the deterministic market. Consider a case with higher renewable integration, where the WPP produces $\tilde{\mathcal{F}} = U(80, 130)$ MW. The energy dispatches, market prices, expected total social costs, and expected profits are presented in Table 4.2. In this case, the stochastic market becomes more favorable for producers, especially the WPP.

Similar to the prior example, we assume that the mean production ($Q^{\mathrm{s}} = 105$ MW) serves as the WPP production forecast in the deterministic market. Since the MO assumes that WPP production will fully meet demand as $Q^{\mathrm{s}} > D = 100$ MW, it assigns the entire demand to the WPP in the DA market. Correspondingly, the market price is set as the marginal production cost of the WPP (0 \$/MWh) as the TPP is not actively producing energy. At this market price, the WPP earns no profit in the DA market. And since there are chances that the WPP faces a production shortfall (e.g., $\omega = 80$ MW), it must buy energy to cover this shortfall in the RT market at a positive price, leading to financial losses. Consequently, the WPP would generally lose money in the deterministic market and may choose to exit.

Table 4.1: Outcomes of the Stochastic and Deterministic Markets ($\tilde{\mathcal{F}} = U(30, 130)$ MW) [1]

| Inc. Bid Prices | Stochastic | | | | | | Deterministic | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $q^{s,*}$ | Price | Prod. Cost | WPP Profit | TPP Profit | Consumer's Cost | $q^{s,*}$ | Price | Prod. Cost | WPP Profit | TPP Profit | Consumer's Cost |
| $\Delta a^{d,+} = \Delta a^{d,-} = 1$ | 80 | 7.3 | 265.5 | 464.5 | 0 | 730 | 80 | 10 | 265.5 | 680.5 | 54 | 1000 |
| $\Delta a^{d,+} = 4, \Delta a^{d,-} = 1$ | 50 | 7.3 | 280.5 | 449.5 | 0 | 730 | 80 | 10 | 303.0 | 643.0 | 54 | 1000 |
| $\Delta a^{d,+} = 0.25, \Delta a^{d,-} = 1$ | 100 | 7.125 | 251.125 | 466.375 | 0 | 712.5 | 80 | 10 | 256.125 | 689.875 | 54 | 1000 |

Table 4.2: Outcomes of the Stochastic and Deterministic Markets ($\tilde{\mathcal{F}} = U(80, 130)$ MW)

| Inc. Bid Prices | Stochastic | | | | | | Deterministic | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $q^{s,*}$ | Price | Prod. Cost | WPP Profit | TPP Profit | Consumer's Cost | $q^{s,*}$ | Price | Prod. Cost | WPP Profit | TPP Profit | Consumer's Cost |
| $\Delta a^{d,+} = \Delta a^{d,-} = 1$ | 100 | 4.4 | 44 | 396 | 0 | 440 | 100 | 0 | 44 | -44 | 0 | 0 |
| $\Delta a^{d,+} = 4, \Delta a^{d,-} = 1$ | 90 | 4.6 | 51 | 409 | 0 | 460 | 100 | 0 | 56 | -56 | 0 | 0 |
| $\Delta a^{d,+} = 0.25, \Delta a^{d,-} = 1$ | 100 | 4.1 | 41 | 369 | 0 | 410 | 100 | 0 | 41 | -41 | 0 | 0 |

[1] Measurement units are hereby clarified: dispatches (MW), prices (\$/MWh), costs (\$), profits (\$).

## 4.2 Incentive Compatibility of the Monopoly Distribution Bidder

Since in this setting only one WPP participates as a renewable energy producer and submit its production distribution, we refer to it as the *monopoly distribution bidder*. Recall from the market optimization problem (3.2) that, if the WPP bids truthfully, by definition, $q^{\mathrm{d},*}(\tilde{\mathcal{F}})$ and $q^{\mathrm{s},*}(\tilde{\mathcal{F}})$ minimize the MO's cost $\mathbb{E}^{\omega\sim\tilde{\mathcal{F}}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]$. Hence, we have

$$\tilde{C}(\tilde{\mathcal{F}}) = \mathbb{E}^{\omega\sim\tilde{\mathcal{F}}}\left[\varphi(q^{\mathrm{d},*}(\tilde{\mathcal{F}}), q^{\mathrm{s},*}(\tilde{\mathcal{F}}), \omega)\right] \leq \mathbb{E}^{\omega\sim\tilde{\mathcal{F}}}\left[\varphi(q^{\mathrm{d},*}(\mathcal{F}), q^{\mathrm{s},*}(\mathcal{F}), \omega)\right] = \tilde{C}(\mathcal{F}), \ \forall \mathcal{F}. \tag{4.4}$$

As an interpretation, the true expected total social cost is minimized when the WPP bids truthfully. Using this result, we discuss if the stochastic market scheme is truth-revealing to the distribution bidder.

### 4.2.1 Low Renewable Penetration

**Theorem 4.2.1.** *Consider the DA market* (3.1) *and RT market* (2.2) *where the TPP submits bids for prices and the WPP submits bids for production distribution $\mathcal{F}$. If the market exhibits* low renewable penetration*, then the true WPP production distribution $\tilde{\mathcal{F}}$ is a* weakly *dominant strategy for maximizing WPP's profit. In other words, the market enjoys a weak truth-revealing property.*

*Proof.* When $\tilde{\phi}(D) = 1$ holds, by bidding truthfully, the WPP can get an expected profit of

$$P^{\mathrm{s}}(\tilde{\mathcal{F}}) = a^{\mathrm{d}}D - \tilde{C}(\tilde{\mathcal{F}}).$$

Due to inequality (4.4), for any bids $\mathcal{F}$ supported on $\mathbb{R}_{\geq 0}$ such that $\phi(D) > c$, it holds that

$$\begin{aligned} P^{\mathrm{s}}(\mathcal{F}) &= a^{\mathrm{d}}D + r^{\mathrm{d}}q^{\mathrm{s},*}(\mathcal{F})\left(\phi(D) - 1\right) - \tilde{C}(\mathcal{F}) \\ &\leq a^{\mathrm{d}}D - \tilde{C}(\tilde{\mathcal{F}}) \\ &= P^{\mathrm{s}}(\tilde{\mathcal{F}}). \end{aligned} \tag{4.5}$$

And for any bids $\mathcal{F}$ such that $\phi(D) \leq c$, as $p^{\mathrm{d}}c \leq a^{\mathrm{d}}$, it holds that

$$\begin{aligned} \tilde{P}(\mathcal{F}) &= p^{\mathrm{d}}D\phi(D) - \tilde{C}(\mathcal{F}) \\ &\leq p^{\mathrm{d}}cD - \tilde{C}(\mathcal{F}) \\ &\leq a^{\mathrm{d}}D - \tilde{C}(\tilde{\mathcal{F}}) \\ &= \tilde{P}(\tilde{\mathcal{F}}). \end{aligned} \tag{4.6}$$

We can then conclude from (4.5) and (4.6) that $\tilde{\mathcal{F}}$ is a dominant strategy for the WPP.

We now show that there exist other distributions that result in the same maximal WPP profit. Any $\mathcal{F}$ satisfying $\phi(D) = 1$ and $\phi^{-1}(c) = \tilde{\phi}^{-1}(c)$ achieve the same optimal dispatch ($q^{\mathrm{s},*}(\mathcal{F}) = \phi^{-1}(c) = q^{\mathrm{s},*}(\tilde{\mathcal{F}})$) and thus the same true expected total social cost $\tilde{C}(\mathcal{F}) = \tilde{C}(\tilde{\mathcal{F}})$. Therefore, they also maximize the WPP's profit:

$$\tilde{P}(\mathcal{F}) = a^{\mathrm{d}}D - \tilde{C}(\mathcal{F}) = a^{\mathrm{d}}D - \tilde{C}(\tilde{\mathcal{F}}) = \tilde{P}(\tilde{\mathcal{F}})$$

Consequently, $\tilde{\mathcal{F}}$ is a *weakly* dominant strategy for maximizing WPP's profit if the WPP admits low renewable penetration. □

### 4.2.2 Moderate and High Renewable Penetration

When the renewable penetration in the market rises such that there are possibilities that the renewables can cover all the demand without any traditional energy producers, the truth-revealing property is lost.

**Theorem 4.2.2.** *If the market exhibits* moderate *or* high renewable penetration *($\tilde{\phi}(D) < 1$), there exists a distribution bid $\mathcal{F}$ such that $P^{\mathrm{s}}(\mathcal{F}) > P^{\mathrm{s}}(\tilde{\mathcal{F}})$.*

*Proof.* We prove theorem 4.2.2 by constructing distribution bids that achieve higher expected profits than a truthful bid under two cases: $c < \tilde{\phi}(D) < 1$ and $\tilde{\phi}(D) \leq c$.

The first case $c < \tilde{\phi}(D) < 1$ infers *moderate renewable penetration*. With a truthful bid $\tilde{\mathcal{F}}$, the WPP achieves the expected total profit of $P^{\mathrm{s}}(\tilde{\mathcal{F}}) = a^{\mathrm{d}}D + r^{\mathrm{d}}D\left(\tilde{\phi}(D) - 1\right) - \tilde{C}(\tilde{\mathcal{F}})$. Construct a non-truthful distribution bid $\mathcal{F}$ with the following PDF:

$$\pi(\omega) = \begin{cases} \tilde{\pi}(\omega), & \omega \leq \tilde{\phi}^{-1}(c) \\ \tilde{\pi}(\omega) + \frac{1-\tilde{\phi}(D)}{D-\tilde{\phi}^{-1}(c)}, & \tilde{\phi}^{-1}(c) < \omega \leq D \\ 0, & \text{otherwise.} \end{cases} \tag{4.7}$$

Compared to the true distribution $\tilde{\mathcal{F}}$, this distribution shifts the likelihood for $\Pr\{\omega > D\}$ to lower productions such that $\phi(D) = 1$ (inferring a low renewable penetration) while maintaining the same optimal WPP dispatch $q^{\mathrm{s},*}(\mathcal{F}) = \phi^{-1}(c) = \tilde{\phi}^{-1}(c) = q^{\mathrm{s},*}(\tilde{\mathcal{F}})$. With the same dispatches, the true expected social cost remains the same, i.e., $\tilde{C}(\mathcal{F}) = \tilde{C}(\tilde{\mathcal{F}})$. The DA market price, however, raises to $\lambda^{\mathrm{DA},*}(\mathcal{F}) = a^{\mathrm{d}} > \lambda^{\mathrm{DA},*}(\tilde{\mathcal{F}})$, and consequently contributes to a higher expected profit

$$\begin{aligned} P^{\mathrm{s}}(\mathcal{F}) &= a^{\mathrm{d}}D + r^{\mathrm{d}}D\left(\tilde{\phi}(D) - 1\right) + r^{\mathrm{d}}q^{\mathrm{s},*}(\mathcal{F})\left(1 - \tilde{\phi}(D)\right) - \tilde{C}(\mathcal{F}) \\ &= a^{\mathrm{d}}D + r^{\mathrm{d}}D\left(\tilde{\phi}(D) - 1\right) + r^{\mathrm{d}}q^{\mathrm{s},*}(\tilde{\mathcal{F}})\left(1 - \tilde{\phi}(D)\right) - \tilde{C}(\tilde{\mathcal{F}}) \\ &= P^{\mathrm{s}}(\tilde{\mathcal{F}}) + r^{\mathrm{d}}q^{\mathrm{s},*}(\tilde{\mathcal{F}})\left(1 - \tilde{\phi}(D)\right) \\ &> P^{\mathrm{s}}(\tilde{\mathcal{F}}). \end{aligned}$$

We then consider the second case $\tilde{\phi}(D) \leq c$ where the market admits *high renewable penetration*. Upon the truthful bid, the WPP's expected total profit is $P^{\mathrm{s}}(\tilde{\mathcal{F}}) = p^{\mathrm{d}}D\tilde{\phi}(D) - \tilde{C}(\tilde{\mathcal{F}})$ We can construct a distribution bid that dominates the true distribution similar to (4.7). Mathematically, due to the fact that $q^{\mathrm{s},*}(\tilde{\mathcal{F}}) = \tilde{\phi}^{-1}(c) = D$, let $\mathcal{F}$ follow the PDF in equation (4.8).

$$\pi(\omega) = \begin{cases} \tilde{\pi}(\omega), & \omega < D \\ \tilde{\pi}(\omega) + \left(1 - \tilde{\phi}(D)\right)\mathrm{Dirac}(\omega - D), & \omega = D \\ 0, & \text{otherwise.} \end{cases} \tag{4.8}$$

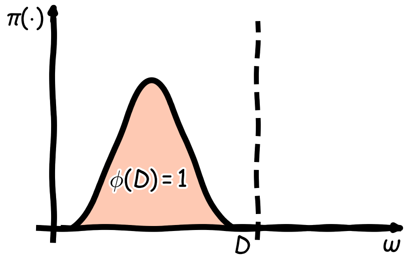
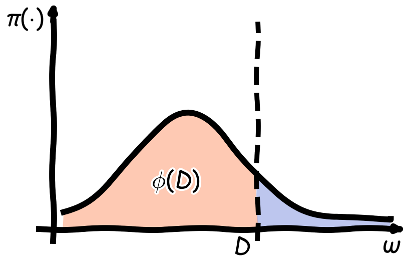
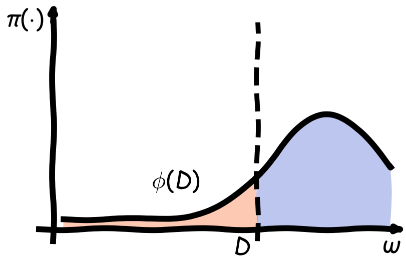
$\mathrm{Dirac}(\cdot)$ denotes the Dirac delta function. In fact, (4.8) characterizes a *mixed distribution* with $\Pr\{\omega = D\} = 1 - \tilde{\phi}(D)$. As in the previous case, this distribution admits $\phi(D) = 1$ and $q^{\mathrm{s},*}(\mathcal{F}) = D = q^{\mathrm{s},*}(\tilde{\mathcal{F}})$, increasing the DA market price to $\lambda^{\mathrm{DA}}(\mathcal{F}) = a^{\mathrm{d}}$ and earns more profit for the WPP:

$$\begin{aligned}
P^{\mathrm{s}}(\mathcal{F}) &= a^{\mathrm{d}}D + r^{\mathrm{d}}D\left(\tilde{\phi}(D) - 1\right) + r^{\mathrm{d}}q^{\mathrm{s},*}(\mathcal{F})\left(1 - \tilde{\phi}(D)\right) - \tilde{C}(\mathcal{F}) \\
&= a^{\mathrm{d}}D + r^{\mathrm{d}}D\left(\tilde{\phi}(D) - 1\right) + r^{\mathrm{d}}D\left(1 - \tilde{\phi}(D)\right) - \tilde{C}(\tilde{\mathcal{F}}) \\
&= a^{\mathrm{d}}D - \tilde{C}(\tilde{\mathcal{F}}) \\
&= P^{\mathrm{s}}(\tilde{\mathcal{F}}) + \left(a^{\mathrm{d}} - p^{\mathrm{d}}\tilde{\phi}(D)\right)D \\
&> P^{\mathrm{s}}(\tilde{\mathcal{F}}).
\end{aligned}$$

Hence, when $\tilde{\phi}(D) < 1$, there exists a distribution bid $\mathcal{F}$ that dominates the truthful bid $\tilde{\mathcal{F}}$. □

Theorem 4.2.2 reveals the lost of the truth-revealing property when the renewable penetration gets higher. The WPP may bid for a distribution that indicates lower renewable penetration to raise the market price and gain more profit. Table 4.3 concludes the results in Theorem 4.2.1 and 4.2.2 about incentive compatibility for the WPP.

Table 4.3: Incentive Compatibility for the WPP under Different Renewable Penetration Levels

| **Renewable Penetration Level** | **Truth-Revealing Property** | **Illustration** |
|---|---|---|
| Low ($\tilde{\phi}(D) = 1$) | Yes |  |
| Moderate ($c < \tilde{\phi}(D) < 1$) | No |  |
| High ($\tilde{\phi}(D) \leq c$) | No |  |

Nevertheless, such results only hold for a monopoly distribution bidder. How their behavior would change when multiple distribution bidders are involved remains an open question for future research.

### 4.2.3 Example

As the first example introduced in Chapter 3.3, let the true WPP production distribution be $\tilde{\mathcal{F}} = U(30, 130)$ MW, and let the bid prices be $a^{\mathrm{d}} = 10$ \$/MWh, $p^{\mathrm{d}} = 11$ \$/MWh, $r^{\mathrm{d}} = 9$ \$/MWh. The optimal shortfall likelihood is $c = 0.5$. The inelastic demand is set as $D = 100$ MW.

For a truthful bid, the WPP admits a *moderate renewable penetration* as $\tilde{\phi}(D) = 0.7 > c$. The optimal WPP dispatch quantity, the corresponding market price, and the expected WPP profit are listed in Table 4.4. As discussed previously in Chapter 4, the WPP has the incentive to claim a lower penetration by strategically bidding an untruthful distribution. For instance, it may bid the distribution $\mathcal{F}$ captured by this following PDF:

$$\pi(\omega) = \begin{cases} 0.01, & 30 \leq \omega \leq 80 \\ 0.025, & 80 < \omega \leq 100 \\ 0, & \text{otherwise.} \end{cases}$$

Figure 4.1 illustrates the shapes of the truthful bid and the strategic bid, respectively.

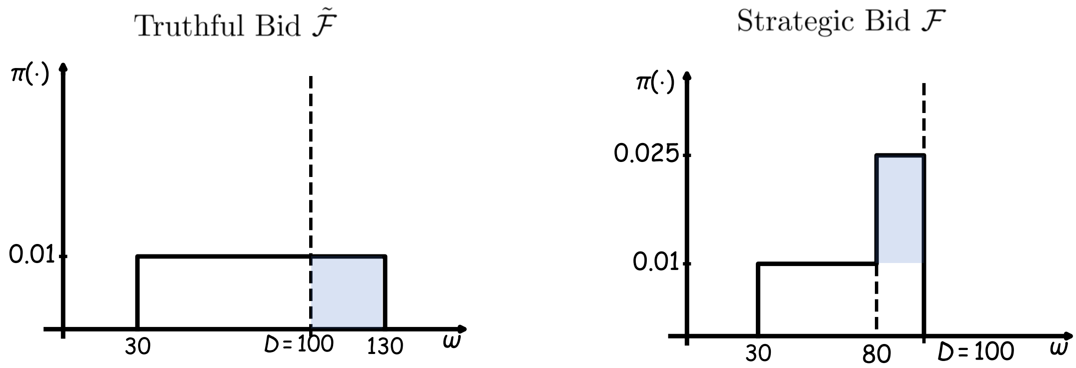



Figure 4.1: Shapes of the Truthful and the Strategic Bids

With this strategic bid, the MO assumes a *low renewable penetration* and rises the market price to 10 \$/MWh, whereas the optimal WPP dispatch remains $\phi^{-1}(c) = \tilde{\phi}^{-1}(c) = 80$ MW. Hence, the WPP makes more profit with the strategic bid.

Table 4.4: Outcomes of the Truthful and the Strategic Bids

| | Penetration | $\phi(D)$ | $q^{\mathrm{s},*}$ (MW) | $\lambda^{\mathrm{DA},*}$ (\$/MWh) | Profit (\$) |
|---|---|---|---|---|---|
| Truthful Bid $\tilde{\mathcal{F}}$ | Moderate | 0.7 | $\tilde{\phi}^{-1}(c) = 80$ | 7.3 | 464.5 |
| Strategic Bid $\mathcal{F}$ | Low | 1 | $\phi^{-1}(c) = 80$ | 10 | 680.5 |

As society increasingly embraces renewable energy, it's clear that while this shift drives down overall production costs, it also grants renewable producers greater influence-potentially tempting them to compromise their integrity. To address the issue, further market rules in the form of rewards and penalties could be designed and implemented, ensuring that renewable producers remain honest and transparent in their bids, keeping the system both efficient and ethical.

# Chapter 5

# Conclusion

In this thesis, we explored optimal market decision-making in the context of increased renewable energy penetration. We analyzed an electricity market with inelastic demand and two types of producers: an aggregated thermal power producer (TPP), representing all traditional energy producers within a competitive market, and a wind power producer (WPP), representing renewable energy. The WPP's stochastic production was modeled using a probability distribution. We applied both deterministic and stochastic market frameworks to this setup, deriving closed-form solutions to optimal dispatches and market prices. Our findings revealed that the behavior of both the market operator (MO) and producers varies depending on the level of renewable penetration, which is determined by the shape of the WPP production distribution as well as the bid prices.

Via multiple examples, we demonstrated key properties of the stochastic electricity market.

- **Optimized Expected Total Social Production Cost**: The stochastic electricity market consistently achieves an optimized expected total social production cost compared to the traditional deterministic electricity market, regardless of the renewable penetration level. This is because the market operator considers the long-term effects of day-ahead (DA) decisions, rather than focusing solely on immediate DA production costs, making the stochastic market inherently forward-looking.
- **Cost Recovery in Expectation**: Both producers achieve cost recovery in expectation in the stochastic market, whereas they may incur losses in certain cases within the deterministic market. This provides the incentive for producers to participate in the stochastic market. Nevertheless, particularly for the TPP, it can generally lose money in the DA market even though this loss is compensated in the real-time market in the long run. Hence, only producers with foresight are willing participants.
- **Incentive Compatibility of the Monopoly Distribution Bidders**: We also introduced the concept of renewable producers acting as distribution bidders, submitting production distributions to the MO. For a single distribution bidder, it will only be **truthful when the market exhibits a low renewable penetration**, i.e., the true renewable energy production never exceeds the total demand. In higher penetration cases, the monopoly distribution bidder has an incentive to submit distributions indicating lower renewable penetration to raise market prices and increase profits.

This raises the crucial question of how to design market mechanisms that ensure the incentive compatibility of distribution bidders with market power, which is an important aspect of future research. A potential solution could involve implementing rewards or penalties based on post-market integrity examinations to encourage truthful distribution bidding. It is also vital to introduce multiple distribution bidders into the market and study their behavior in the market.

Additionally, while this thesis assumes producers bid constant prices, real-world scenarios often involve more complex bidding strategies. Future research should extend this model to incorporate more general bid structures, such as linear or piecewise linear price functions. Further exploration of elastic demand, time-wise and location-wise renewable production correlations, and transmission constraints/costs would also be valuable in bridging the gap between theoretical models and real-world applications.

# Reference


[1] Klaus Mayer and Stefan Trück. "Electricity markets around the world". In: *Journal of Commodity Markets* 9 (2018), pp. 77–100.

[2] D Kirschen and G Strbac. *Fundamentals of Power System Economics*. English. United Kingdom: John Wiley & Sons Ltd, 2004.

[3] Juan M. Morales et al. *Integrating Renewables in Electricity Markets: Operational Problems*. en. Vol. 205. International Series in Operations Research & Management Science. Boston, MA: Springer, 2014.

[4] Victor M Zavala et al. "A stochastic electricity market clearing formulation with consistent pricing properties". In: *Operations Research* 65.3 (2017), pp. 557–576.

[5] Energinet. *Energy In Time Energinet Strategy 2022*. Tech. rep. Energinet, 2022.

[6] Geoffrey Pritchard, Golbon Zakeri, and Andrew Philpott. "A single-settlement, energy-only electric power market for unpredictable and intermittent participants". In: *Operations research* 58.4-part-2 (2010), pp. 1210–1219.

[7] Steven Wong and J David Fuller. "Pricing energy and reserves using stochastic optimization in an alternative electricity market". In: *IEEE Transactions on Power Systems* 22.2 (2007), pp. 631–638.

[8] Juan M Morales et al. "Pricing electricity in pools with wind producers". In: *IEEE Transactions on Power Systems* 27.3 (2012), pp. 1366–1376.

[9] Juan M Morales et al. "Electricity market clearing with improved scheduling of stochastic production". In: *European Journal of Operational Research* 235.3 (2014), pp. 765–774.

[10] Jalal Kazempour, Pierre Pinson, and Benjamin F Hobbs. "A stochastic market design with revenue adequacy and cost recovery by scenario: Benefits and costs". In: *IEEE Transactions on Power Systems* 33.4 (2018), pp. 3531–3545.

[11] Xiaolong Kuang et al. "Pricing chance constraints in electricity markets". In: *IEEE Transactions on Power Systems* 33.4 (2018), pp. 4634–4636.

[12] Yury Dvorkin. "A chance-constrained stochastic electricity market". In: *IEEE Transactions on Power Systems* 35.4 (2019), pp. 2993–3003.

[13] Robert Mieth, Jip Kim, and Yury Dvorkin. "Risk-and variance-aware electricity pricing". In: *Electric Power Systems Research* 189 (2020), p. 106804.

[14] Robert Mieth, Matt Roveto, and Yury Dvorkin. "Risk trading in a chance-constrained stochastic electricity market". In: *IEEE Control Systems Letters* 5.1 (2020), pp. 199–204.

[15] Adriano Arrigo et al. "Wasserstein distributionally robust chance-constrained optimization for energy and reserve dispatch: An exact and physically-bounded formulation". In: *European Journal of Operational Research* 296.1 (2022), pp. 304–322.

[16] Stephen Boyd and Lieven Vandenberghe. *Convex optimization.* Cambridge university press, 2004.

[17] Anton J Kleywegt and Alexander Shapiro. “Stochastic optimization”. In: *Handbook of industrial engineering* (2001), pp. 2625–2649.

[18] Jean-Baptiste Hiriart-Urruty and Claude Lemaréchal. *Convex Analysis and Minimization Algorithms I.* Vol. 305. Grundlehren der mathematischen Wissenschaften. Springer, 1993.

[19] Boris Mordukhovich and Nguyen Mau Nam. *An Easy Path to Convex Analysis and Applications.* Synthesis Lectures on Mathematics & Statistics. Springer International Publishing, 2023.

[20] R Tyrrell Rockafellar and Roger J-B Wets. *Variational analysis.* Vol. 317. Springer Science & Business Media, 2009.

# Appendix A

# Optimality Conditions for Nonsmooth-Convex Problems

For a convex optimization problem, if the objective function and constraints are everywhere differentiable, the KKT conditions are both necessary and sufficient for optimality [16, Chapter 5.5]. However, for a nonsmooth function whose gradient is not accessible at some points within the domain, the KKT conditions might not apply. As an alternative, this chapter introduces the optimality conditions for nonsmooth convex problems using *subdifferential*.

## A.1 Preliminaries

### A.1.1 Subdifferential

We first introduce the definition of subdifferential.

**Definition A.1.1** ([18, Definition VI.1.2.1])**.** *The subdifferential of a function $f : \mathbb{R}^n \to \mathbb{R}$ at $x \in \mathrm{dom} f$ is the set of vectors $v \in \mathbb{R}^n$ satisfying*

$$f(y) \geq f(x) + v^\top (y - x) \quad \forall y \in \mathrm{dom} f.$$

*Every such vector is called a* subgradient *of $f$ at $x$.*

This definition infers that a vector $v$ is a subgradient of $f$ at $x$ if the affine function $f(x)+v^\top(y-x)$ is a global underestimator of $f$. We refer to [18, Chapter VI] for further interpretations and properties of subgradients. A subgradient can exist even if $f$ is not differentiable at $x$. Specially, if $f$ is convex and differentiable, then its gradient at $x$ is a subgradient.

We denote the subdifferential of $f$ at $x$ by $\partial_x f(x)$. Similar to ordinary differential calculus, the subdifferential calculus rules are essential for the computation and have been studied in the literature. The results are typically characterized in two levels. For the "weak" calculus, the rules typically find one subgradient for the constructed function even if multiple subgradients exist. The "strong" calculus, on the other hand, produces the complete set of subgradients. One of the fundamental results of "strong" subdifferential calculus targets positive combinations of convex functions as introduced in Lemma A.1.2.

**Lemma A.1.2** ([18, Theorem VI.4.1.1])**.** *Let $t_1, t_2 \in \mathbb{R}_{>0}$. For functions $f_1 : \mathbb{R}^n \to \mathbb{R}$ and $f_2 : \mathbb{R}^n \to \mathbb{R}$, it holds that*

$$\partial_x (t_1 f_1 + t_2 f_2)(x) = t_1 \partial_x f_1(x) + t_2 \partial_x f_2(x) \quad \forall x \in \mathrm{dom}(t_1 f_1 + t_2 f_2).$$

The positive combination property can be extended to expectations (if exist) as a "weak" result.

**Proposition A.1.3** (Weak Subdifferential Calculus Rule for Expectations). *Let $u \sim \mathcal{F}$ be a random variable. For a function $f(x) = \mathbb{E}^u[h(x,u)]$ where $h : (\mathbb{R}^n, \mathcal{F}) \to \mathbb{R}$ is convex in $x$ for all $u$, by finding a function $g : \mathcal{F} \to \mathbb{R}^n$ such that $g(u) \in \partial_x h(x,u), \forall u$, it holds that $\mathbb{E}^u[g(u)] \in \partial_x f(x)$.*

*Proof.* For all $y \in \operatorname{dom} h$, as $g(u) \in \partial_x h(x,u), \forall u$, we have $h(y,u) \geq h(x,u) + g(u)^\top (y-x)$. Due to convexity of $h(x,u)$ in $x$, it holds that

$$\begin{aligned} f(y) &= \mathbb{E}^u[h(y,u)] \\ &\geq \mathbb{E}^u\left[h(x,u) + g(u)^\top (y-x)\right] \\ &= f(x) + \mathbb{E}^u[g(u)]^\top (y-x), \end{aligned}$$

which infers $\mathbb{E}^u[g(u)] \in \partial_x f(x)$. □

Further subdifferential calculus rules for pre-composition and post-composition have also been studied. Here we provide a particular result for the pointwise maximum function, which was applied previously in this thesis.

**Lemma A.1.4** ([18, Corollary VI.4.3.2]). *Let $f_1, f_2, \ldots, f_m$ be $m$ convex functions mapping from $\mathbb{R}^n$ to $\mathbb{R}$. Define the pointwise maximum function as $f(x) := \max\{f_1(x), f_2(x), \ldots, f_m(x)\}$. Denote the active index set by $I(x) := \{i \mid f_i(x) = f(x)\}$. Then we have*

$$\partial_x f(x) = \operatorname{Conv}\{\partial_x f_i(x) \mid i \in I(x)\}.$$

### A.1.2 Normal Cone

**Definition A.1.5** ([18, Definition III.5.2.3]). *Let $S \subset \mathbb{R}^n$ be nonempty with $x \in S$. The* normal cone *to $S$ at $x$ is*

$$N_S(x) := \{v \in \mathbb{R}^n \mid v^\top (y-x) \leq 0, \ \forall y \in S\}.$$

For the intersection of several sets, rather than finding its normal cone by its definition, sometimes it is easier to use the *normal cone intersection rule* to compute.

**Lemma A.1.6** ([18, Proposition III.5.3.1.iv]). *Let $S_1, S_2 \subset \mathbb{R}^n$ be nonempty with $x \in S_1 \cap S_2$. It holds that*

$$N_{S_1 \cap S_2}(x) \supset N_{S_1}(x) + N_{S_2}(x).$$

In fact, if the intersection of *relative interior* of both sets $S_1$ and $S_2$ is nonempty, we can extend the result to

$$N_{S_1 \cap S_2}(x) = N_{S_1}(x) + N_{S_2}(x).$$

We refer to [19, Theorem 2.56] for a detailed proof.

## A.2 Optimality Conditions

**Lemma A.2.1** (Sufficient Optimality Conditions in [20, Theorem 8.15]). *Consider a problem of minimizing a convex function $f : \mathbb{R}^n \to \mathbb{R}$ over a convex set $S \subset \mathbb{R}^n$. $x^* \in S$ is globally optimal if it holds that*

$$0 \in \partial_x f(x^*) + N_S(x^*). \tag{A.1}$$

*In other words,* (A.1) *provides a sufficient optimality condition for general convex optimization problems, whether they are smooth or not.*

[20, Theorem 8.15] also states the necessary optimality conditions and the corresponding constraint qualification for which to hold, for more general optimization problems (not limited to convex optimization problems).

# Appendix B

# Linear Marginal Costs

Previous market analysis in this thesis was conducted upon *constant marginal costs* (submitted via producers' price bids), which lead to linear production costs for the producers. However, this is a rather unrealistic assumption as a constant value can hardly describe the actual marginal production costs. In practice, many existing electricity markets allow producers and consumers to submit bids with piecewise linear marginal price functions, which infer (piecewise) quadratic production costs. Hence, it is vital to extend the model to quadratic production cost and further analyze the market outcomes.

## B.1 Modeling Using Linear Marginal Costs

We consider the same electricity market with inelastic demand $D \in \mathbb{R}_{\geq 0}$. One truthful thermal power producer (TPP) and a wind power producer (WPP) with zero marginal production cost participate in the market as producers.

We first discuss the deterministic market. In the DA market, the marginal production cost of the TPP is modeled as

$$a(q^{\mathrm{d}}) = \alpha q^{\mathrm{d}},$$

where $\alpha \in \mathbb{R}_{\geq 0}$. As the TPP production increases, the marginal production cost increases, which reflects the fact that generators with lower marginal production costs are preferred over those with high marginal production costs. Consequently, given the TPP DA production $q^{\mathrm{d}}$, its DA production cost is $\frac{1}{2}\alpha {q^{\mathrm{d}}}^2$. We can then model the DA social welfare optimization problem (production cost minimization) as follows:

$$\begin{aligned} C^{\mathrm{DA}} = \min_{q^{\mathrm{d}}, q^{\mathrm{s}}} \quad & \frac{1}{2}\alpha {q^{\mathrm{d}}}^2 && \text{(B.1a)} \\ \text{s.t.} \quad & q^{\mathrm{d}} + q^{\mathrm{s}} = D, \quad (\lambda^{\mathrm{DA}}) && \text{(B.1b)} \\ & 0 \leq q^{\mathrm{d}} && \text{(B.1c)} \\ & 0 \leq q^{\mathrm{s}} \leq Q^{\mathrm{s}}. && \text{(B.1d)} \end{aligned}$$

Similar to market (2.1), (B.1b) is the demand balancing condition. The corresponding dual variable $\lambda^{\mathrm{DA}}$ coincides with the market clearing price. As we assume a very large TPP production capacity, (B.1c) and (B.1d) limit the dispatch quantity, where $Q^{\mathrm{s}}$ denotes the WPP production estimation.

As for the RT market, the TPP sells additional production if the WPP produces a shortfall and buys cheap electricity if the WPP surplus happens. When selling, the TPP's bid reflects its RT marginal production cost, which is typically higher than the DA market as it means additional production. Thus, when additional production of the TPP is required, we assume the RT marginal production cost to be

$$\alpha q^{\mathrm{d}} + \rho[\delta^{\mathrm{d}}]_{+}, \tag{B.2}$$

where $\rho \in \mathbb{R}_{\geq 0}$. On the other hand, when the TPP buys energy, its bid reflects the cost that it can recover from producing less, which is modeled as follows:

$$\alpha q^{\mathrm{d}} - \gamma[\delta^{\mathrm{d}}]_{-}, \tag{B.3}$$

with $\gamma \in \mathbb{R}_{\geq 0}$. We refer to it as the *marginal recoverable cost*. The RT optimization problem is revised from market (2.2) with a newly defined cost as follows:

$$\begin{aligned}
C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) = \min_{\delta^{\mathrm{d}}, \delta^{\mathrm{s}}} \quad & \underbrace{\frac{1}{2}\rho[\delta^{\mathrm{d}}]_{+}^{2} + \alpha q^{\mathrm{d}}[\delta^{\mathrm{d}}]_{+}}_{\text{additional production cost}} + \underbrace{\frac{1}{2}\gamma[\delta^{\mathrm{d}}]_{-}^{2} - \alpha q^{\mathrm{d}}[\delta^{\mathrm{d}}]_{-}}_{\text{recovered cost}} && \text{(B.4a)} \\
\text{s.t.} \quad & \delta^{\mathrm{d}} + \delta^{\mathrm{s}} = 0, \quad (\lambda^{\mathrm{RT}}) && \text{(B.4b)} \\
& 0 \leq q^{\mathrm{d}} + \delta^{\mathrm{d}} && \text{(B.4c)} \\
& 0 \leq q^{\mathrm{s}} + \delta^{\mathrm{s}} \leq \omega. && \text{(B.4d)}
\end{aligned}$$

Recall that $\omega$ denotes the realization of the WPP production.

Similarly, we also revise the production cost function of the stochastic DA market from (3.1) to

$$\begin{aligned}
C(\mathcal{F}) = \min_{q^{\mathrm{d}}, q^{\mathrm{s}}} \quad & \frac{1}{2}\alpha {q^{\mathrm{d}}}^{2} + \mathbb{E}^{\omega\sim\mathcal{F}}\left[C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] && \text{(B.5a)} \\
\text{s.t.} \quad & q^{\mathrm{d}} + q^{\mathrm{s}} = D, \quad (\lambda^{\mathrm{DA}}) && \text{(B.5b)} \\
& q^{\mathrm{d}}, q^{\mathrm{s}} \geq 0, && \text{(B.5c)}
\end{aligned}$$

where $C^{\mathrm{RT}}(\cdot,\cdot,\cdot)$ is the optimal RT cost defined in (B.4).

## B.2 Results

We focus on analyzing the stochastic DA market with linear marginal costs in (B.5) and compare its outcomes to the case with constant marginal costs.

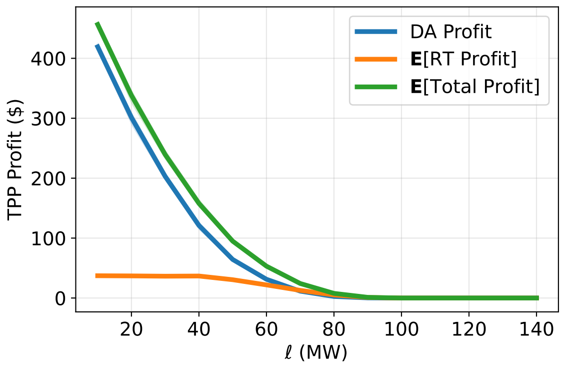
This new problem is harder to solve. Firstly, with linear marginal costs, the problem becomes *nonconvex*. As the TPP's RT marginal production cost (B.2) and marginal recoverable cost (B.3) directly depend on the DA WPP dispatch $q^{\mathrm{d}}$, in the second stage (the RT market) of (B.5), there exist bilinear terms $\alpha q^{\mathrm{d}}[\delta^{\mathrm{d}}]_{+}$ and $\alpha q^{\mathrm{d}}[\delta^{\mathrm{d}}]_{-}$. Secondly, a general closed-form solution for problem (B.5) *does not exist* for any production distribution $\mathcal{F}$ anymore.

Hence, to compare the market outcomes, we randomly sample a certain number of scenarios from the distribution $\mathcal{F}$ such that we can reformulate the problem using scenario-based methods (e.g., as in [4]) and solve it using a nonlinear solver. All numerical experiments are conducted using IPOPT [1] within a Python 3.10 environment on a Windows 11 64-bit workstation with a 3.20 GHz AMD Ryzen 7-5800H CPU and 16.0GB RAM.

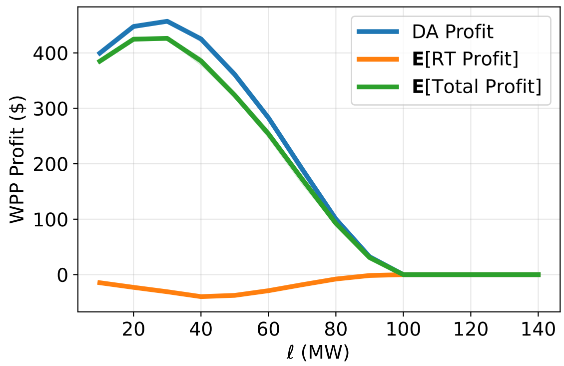
[1]https://coin-or.github.io/Ipopt/

### B.2.1 Cost Revocery

Let the inelastic demand be $D = 100$ MW. And let the marginal cost slopes be $\alpha = 0.2, \rho = 0.4, \gamma = 0.3$. We numerically examine if the market with linear marginal costs enjoys cost recovery for both producers, under different production distribution $\mathcal{F}$. Here we mainly show the results under uniform distributions $U(\ell, \ell+50)$, where we take different values for $\ell$ in the range $[10, 140]$ MW. A larger $\ell$ represents a higher renewable penetration. When $\ell \geq 100$ MW, the WPP is able to cover all the demand itself. Figure B.1 illustrates the expected profits (including the DA profits and expected RT profits) for both producers.




(a) TPP




(b) WPP

Figure B.1: Producers' profits in the market with linear marginal costs ($\mathcal{F} = U(\ell, \ell + 50)$)

Recall that in the case of constant marginal costs, while the WPP makes a nonnegative expected total profit, the TPP's expected total profit always equals zero because it typically loses money on the DA market. Excitingly, with linear marginal costs, which are closer to reality, the TPP is profitable. The linear marginal costs enable a more reasonable market clearing price and hence TPP does not necessarily lose money on the DA market. In fact, this property is provable for any production distribution $\mathcal{F}$, not limited to certain numerical examples. For simplicity, we first assume a continuous probability density function for the production distribution and prove the property.

**Theorem B.2.1.** *Consider market* (B.5). *Assume that the WPP's production distribution* $\tilde{\mathcal{F}}$ *has a continuous probability density function* $\pi(\cdot)$. *If the WPP bids truthfully (*$\mathcal{F} = \tilde{\mathcal{F}}$*), the TPP makes a nonnegative expected total profit.*

*Proof.* Although the entire two-stage problem is nonconvex, given the DA dispatches $q^{\mathrm{d}}$ and $q^{\mathrm{s}}$, the second stage (the RT market) is convex and can be solved in closed form. The optimal RT production cost, dispatches, and market price are listed in Table B.1. Note that the TPP RT

| Production Realization | $0 \leq \omega \leq q^{\mathrm{s}}$ (WPP Shortfall) | $q^{\mathrm{s}} < \omega \leq q^{\mathrm{s}} + \frac{\alpha}{\gamma} q^{\mathrm{d}}$ (WPP Surplus) | $q^{\mathrm{s}} + \frac{\alpha}{\gamma} q^{\mathrm{d}} < \omega$ (WPP Surplus) |
|---|---|---|---|
| $C^{\mathrm{RT}}$ | $\frac{1}{2}\rho(q^{\mathrm{s}} - \omega)^2 + \alpha q^{\mathrm{d}}(q^{\mathrm{s}} - \omega)$ | $\frac{1}{2}\gamma(q^{\mathrm{s}} - \omega)^2 + \alpha q^{\mathrm{d}}(q^{\mathrm{s}} - \omega)$ | $-\frac{1}{2}\frac{\alpha^2}{\gamma} q^{\mathrm{d}2}$ |
| $\delta^{\mathrm{d},*}, \delta^{\mathrm{s},*}$ | $\delta^{\mathrm{d},*} = -\delta^{\mathrm{s},*} = q^{\mathrm{s}} - \omega$ | $\delta^{\mathrm{d},*} = -\delta^{\mathrm{s},*} = q^{\mathrm{s}} - \omega$ | $\delta^{\mathrm{d},*} = -\delta^{\mathrm{s},*} = -\frac{\alpha}{\gamma} q^{\mathrm{d}}$ |
| $\lambda^{\mathrm{RT},*}$ | $\rho(q^{\mathrm{s}} - \omega) + \alpha q^{\mathrm{d}}$ | $\gamma(q^{\mathrm{s}} - \omega) + \alpha q^{\mathrm{d}}$ | 0 |
| TPP RT Profit | $\frac{1}{2}\rho(q^{\mathrm{s}} - \omega)^2$ | $\frac{1}{2}\gamma(q^{\mathrm{s}} - \omega)^2$ | $\frac{1}{2}\frac{\alpha^2}{\gamma} q^{\mathrm{d}2}$ |

Table B.1: Closed-form solutions for the RT market (B.4) with linear marginal costs profit is calculated using $\lambda^{\mathrm{RT},*}\delta^{\mathrm{d},*} - C^{\mathrm{RT}}$. When the WPP produces a shortfall compared to the DA schedule ($\omega \leq q^{\mathrm{s}}$), the TPP takes the responsibility of producing the part in shortage. On the other hand, the TPP benefits from buying cheap energy from the WPP when there is a surplus, with its maximum willing quantity of buying being $\frac{\alpha}{\gamma}q^{\mathrm{d}}$, as it only recovers costs within this quantity.

We are then able to write down the optimality conditions for the entire two-stage problem with this optimal solution for the second stage. Denote that $\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega) = \frac{1}{2}\alpha {q^{\mathrm{d}}}^2 + C^{\mathrm{RT}}(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)$, then the expected total social production cost in (B.5a) can be rewritten as $\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]$. Using the solutions in Table B.1, we can expand it as follows:

$$
\begin{aligned}
\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] = \frac{1}{2}\alpha {q^{\mathrm{d}}}^2 &+ \int_0^{q^{\mathrm{s}}} \left(\frac{1}{2}\rho(q^{\mathrm{s}}-\omega)^2 + \alpha q^{\mathrm{d}}(q^{\mathrm{s}}-\omega)\right)\pi(\omega)\mathrm{d}\omega \\
&+ \int_{q^{\mathrm{s}}}^{\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}}} \left(\frac{1}{2}\gamma(q^{\mathrm{s}}-\omega)^2 + \alpha q^{\mathrm{d}}(q^{\mathrm{s}}-\omega)\right)\pi(\omega)\mathrm{d}\omega \\
&- \frac{1}{2}\frac{\alpha^2}{\gamma}{q^{\mathrm{d}}}^2\left(1 - \phi(\frac{\alpha}{\gamma}q^{\mathrm{d}} + q^{\mathrm{s}})\right).
\end{aligned}
$$

Then we have

$$
\begin{aligned}
\nabla_{q^{\mathrm{d}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] &= \alpha q^{\mathrm{d}} + \int_0^{\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}}} \alpha(q^{\mathrm{s}}-\omega)\pi(\omega)\mathrm{d}\omega + \frac{\alpha^2}{\gamma}q^{\mathrm{d}}\left(\phi(\frac{\alpha}{\gamma}q^{\mathrm{d}} + q^{\mathrm{s}}) - 1\right) \\
\nabla_{q^{\mathrm{s}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] &= \int_0^{q^{\mathrm{s}}} \rho(q^{\mathrm{s}}-\omega)\pi(\omega)\mathrm{d}\omega + \int_{q^{\mathrm{s}}}^{\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}}} \gamma(q^{\mathrm{s}}-\omega)\pi(\omega)\mathrm{d}\omega \\
&\quad + \int_0^{\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}}} \alpha q^{\mathrm{d}}\pi(\omega)\mathrm{d}\omega
\end{aligned}
$$

With the Lagrangian of problem (B.5) being

$$
\mathcal{L}(q^{\mathrm{d}}, q^{\mathrm{s}}, \lambda^{\mathrm{DA}}, \lambda^{\mathrm{d}}, \lambda^{\mathrm{s}}) = \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right] + \lambda^{\mathrm{DA}}(D - q^{\mathrm{d}} - q^{\mathrm{s}}) - \lambda^{\mathrm{d}}q^{\mathrm{d}} - \lambda^{\mathrm{s}}q^{\mathrm{s}},
$$

applying Leibniz Integral Rule, we can write down the KKT conditions as follows:

$$
\nabla_{q^{\mathrm{d}}}\mathcal{L} = \nabla_{q^{\mathrm{d}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d},*}, q^{\mathrm{s},*}, \omega)\right] - \lambda^{\mathrm{DA},*} - \lambda^{\mathrm{d},*} = 0 \tag{B.6a}
$$

$$
\nabla_{q^{\mathrm{s}}}\mathcal{L} = \nabla_{q^{\mathrm{s}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d},*}, q^{\mathrm{s},*}, \omega)\right] - \lambda^{\mathrm{DA},*} - \lambda^{\mathrm{s},*} = 0 \tag{B.6b}
$$

$$
q^{\mathrm{d},*} + q^{\mathrm{s},*} = D \tag{B.6c}
$$

$$
0 \leq q^{\mathrm{d},*} \perp \lambda^{\mathrm{d},*} \geq 0 \tag{B.6d}
$$

$$
0 \leq q^{\mathrm{s},*} \perp \lambda^{\mathrm{s},*} \geq 0 \tag{B.6e}
$$

For simplicity, we omit the superscript $*$ for the rest of the proof. More specifically, we use $q^{\mathrm{d}}$, $q^{\mathrm{s}}$, $\lambda^{\mathrm{DA}}$, $\lambda^{\mathrm{d}}$, and $\lambda^{\mathrm{s}}$ to represent the optimal solutions $q^{\mathrm{d},*}$, $q^{\mathrm{s},*}$, $\lambda^{\mathrm{DA},*}$, $\lambda^{\mathrm{d},*}$, and $\lambda^{\mathrm{s},*}$, respectively.

- **Case 1:** $q^{\mathrm{d}} > 0$

According to complementary slackness in (B.6d), we have $\lambda^{\mathrm{d}} = 0$, and hence the market clearing price is $\lambda^{\mathrm{DA}} = \nabla_{q^{\mathrm{d}}}\mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}}, q^{\mathrm{s}}, \omega)\right]$. Then the TPP DA profit is

$$
\begin{aligned}
P^{\mathrm{DA,d}} &= \lambda^{\mathrm{DA}}q^{\mathrm{d}} - \frac{1}{2}\alpha {q^{\mathrm{d}}}^2 \\
&= \frac{1}{2}\alpha {q^{\mathrm{d}}}^2 + \int_0^{\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}}} \alpha q^{\mathrm{d}}(q^{\mathrm{s}}-\omega)\pi(\omega)\mathrm{d}\omega + \frac{\alpha^2}{\gamma}q^{\mathrm{d}}\left(\phi(\frac{\alpha}{\gamma}q^{\mathrm{d}} + q^{\mathrm{s}}) - 1\right),
\end{aligned}
$$

and the expected TPP RT profit is

$$P^{\mathrm{RT,d}} = \int_0^{q^{\mathrm{s}}} \frac{1}{2}\rho(q^{\mathrm{s}}-\omega)^2\pi(\omega)\mathrm{d}\omega + \int_{q^{\mathrm{s}}}^{\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}}} \frac{1}{2}\gamma(q^{\mathrm{s}}-\omega)^2\pi(\omega)\mathrm{d}\omega + \frac{1}{2}\frac{\alpha^2}{\gamma}q^{\mathrm{d}}\left(1-\phi(\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}})\right).$$

Thus, the expected total TPP profit is

$$\begin{aligned} P^{\mathrm{d}} &= P^{\mathrm{DA,d}} + P^{\mathrm{RT,d}} \\ &= \frac{1}{2}\alpha {q^{\mathrm{d}}}^2 + \int_0^{q^{\mathrm{s}}} \left(\frac{1}{2}\rho(q^{\mathrm{s}}-\omega)^2 + \alpha q^{\mathrm{d}}(q^{\mathrm{s}}-\omega)\right)\pi(\omega)\mathrm{d}\omega \\ &\quad + \int_{q^{\mathrm{s}}}^{\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}}} \left(\frac{1}{2}\gamma(q^{\mathrm{s}}-\omega)^2 + \alpha q^{\mathrm{d}}(q^{\mathrm{s}}-\omega)\right)\pi(\omega)\mathrm{d}\omega \\ &\quad + \frac{1}{2}\frac{\alpha^2}{\gamma}q^{\mathrm{d}}\left(\phi(\frac{\alpha}{\gamma}q^{\mathrm{d}}+q^{\mathrm{s}})-1\right) \\ &= \mathbb{E}^{\omega\sim\mathcal{F}}\left[\varphi(q^{\mathrm{d}},q^{\mathrm{s}},\omega)\right] \\ &\geq 0. \end{aligned} \tag{B.7}$$

Equation (B.7) shows that the overall TPP profit equals the total social production cost and thus is nonnegative.

- **Case 2:** $q^{\mathrm{d}} = 0$

In this case, $q^{\mathrm{s}} = D - q^{\mathrm{d}} = D$. Since the DA TPP dispatch is zero, the TPP makes zero profits on the DA market, i.e., $P^{\mathrm{DA,d}} = 0$. And the expected TPP RT profit is

$$P^{\mathrm{RT,d}} = \int_0^{D} \frac{1}{2}\rho(D-\omega)^2\pi(\omega)\mathrm{d}\omega \geq 0.$$

Hence, we have

$$P^{\mathrm{d}} = P^{\mathrm{DA,d}} + P^{\mathrm{RT,d}} = P^{\mathrm{RT,d}} \geq 0. \tag{B.8}$$

In conclusion, according to equation (B.7) and equation (B.8), the TPP makes a nonnegative expected total profit. □